3 More about Graphs of Functions

Review Exercise 3 (p. 3.5)

1. (a) $f(3) = 2(3)^2 - 3(3) - 4$
$= \underline{\underline{5}}$

(b) $f\left(\frac{1}{2}\right) = 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{2}\right) - 4$
$= \underline{\underline{-5}}$

2. (a) $g(a-1) = (a-1)^2 + 4(a-1)$
$= a^2 - 2a + 1 + 4a - 4$
$= \underline{\underline{a^2 + 2a - 3}}$

(b) $g(2b+1) = (2b+1)^2 + 4(2b+1)$
$= 4b^2 + 4b + 1 + 8b + 4$
$= \underline{\underline{4b^2 + 12b + 5}}$

3. (a) $\because \quad f(2) = 6$
$k(2) - 2 = 6$
$\therefore \quad 2k = 8$
$k = \underline{\underline{4}}$

(b) From (a), we have $f(x) = 4x - 2$.
$\because \quad f(t) = -6$
$4t - 2 = -6$
$\therefore \quad 4t = -4$
$t = \underline{\underline{-1}}$

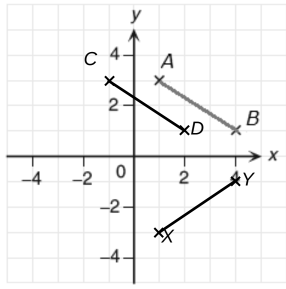
4. (a) The coordinates of $Q = (3, 2+2) = \underline{\underline{(3, 4)}}$

(b) The coordinates of $R = \underline{\underline{(3, -2)}}$

5. The coordinates of $Q = (-(-5+4), 2) = \underline{\underline{(1, 2)}}$

6. Let (x, y) be the coordinates of P.
The coordinates of $Q = (x - 5, -y) = (4, -7)$
$\therefore \quad x - 5 = 4$
$x = 9$
and $\quad -y = -7$
$y = 7$
$\therefore$ The coordinates of $P = \underline{\underline{(9, 7)}}$

7. (a), (b)



Activity

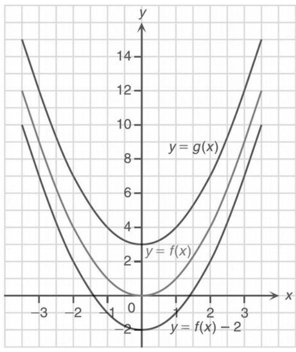
Activity 3.1 (p. 3.41)

1. $g(x) = f(x) + 3$
$= \underline{\underline{x^2 + 3}}$

2.

x	–3	–2	–1	0	1	2	3
$f(x)$	9	<u>4</u>	<u>1</u>	0	1	<u>4</u>	<u>9</u>
$g(x)$	<u>12</u>	<u>7</u>	4	<u>3</u>	<u>4</u>	7	12

3, 6(b)



4. Yes

5. The graph of $y = f(x) + 3$ can be obtained by translating the graph of $y = f(x)$ <u>upwards</u> by <u>3</u> units.

6. (a) Yes

(b) The graph of $y = f(x) - 2$ can be obtained by translating the graph of $y = f(x)$ downwards by 2 units.

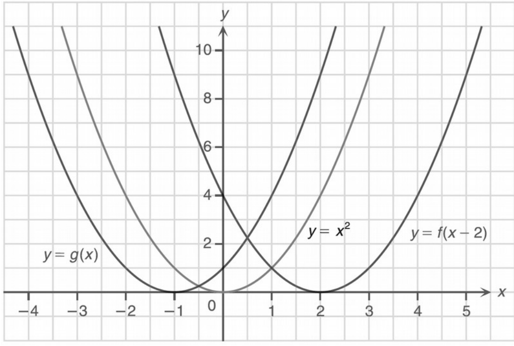
Activity 3.2 (p. 3.45)

1. $g(x) = f(x+1)$
$= \underline{\underline{(x+1)^2}}$

2.

x	–4	–3	–2	–1	0	1	2	3
$f(x)$	...	9	<u>4</u>	<u>1</u>	0	1	<u>4</u>	<u>9</u>
$g(x)$	9	4	<u>1</u>	<u>0</u>	<u>1</u>	4	<u>9</u>	...

3, 6 (b)



4. Yes

5. The graph of $y = f(x+1)$ can be obtained by translating the graph of $y = f(x)$ <u>leftwards</u> by <u> 1 </u> unit.

6. **(a)** Yes

(b) The graph of $y = f(x-2)$ can be obtained by translating the graph of $y = f(x)$ rightwards by 2 units.

Maths Dialogue

Maths Dialogue (p. 3.59)

1.
$$\begin{aligned} g(x) &= 4x^2 - 4 \\ &= 4(x^2 - 1) \\ &= 4f(x) \end{aligned}$$

∴ The graph of $y = f(x)$ is enlarged along the y-axis to 4 times the original to give the graph of $y = g(x)$.

∴ The graph of $y = g(x)$ can be obtained by Ken's approach.

Suppose the graph of $y = g(x)$ can be obtained by Angel's approach.

i.e. $g(x) = f(kx)$ for some constant k.

$$\begin{aligned} g(x) &= 4x^2 - 4 \\ (kx)^2 - 1 &= 4x^2 - 4 \\ k^2x^2 &= 4x^2 - 3 \\ k^2 &= 4 - \frac{3}{x^2} \\ k &= \pm\sqrt{4 - \frac{3}{x^2}}, \text{ which are not constant} \end{aligned}$$

∴ The graph of $y = g(x)$ cannot be obtained by Angel's approach.

2.
$$\begin{aligned} g(x) &= x^2 + x \\ &= (-x)^2 - (-x) \\ &= f(-x) \end{aligned}$$

∴ The graph of $y = g(x)$ can be obtained by reflecting the graph of $y = f(x)$ about the y-axis.

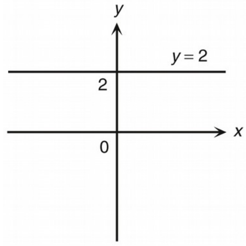
$$\begin{aligned} g(x) &= x^2 + x \\ &= x(x+1) \\ &= (x+1-1)(x+1) \\ &= (x+1)^2 - (x+1) \\ &= f(x+1) \end{aligned}$$

∴ The graph of $y = g(x)$ can be obtained by translating the graph of $y = f(x)$ leftwards by 1 unit.

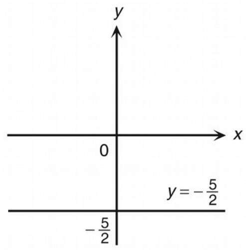
Classwork

Classwork (p. 3.6)

(a)

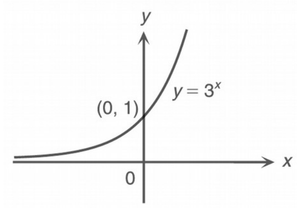


(b)

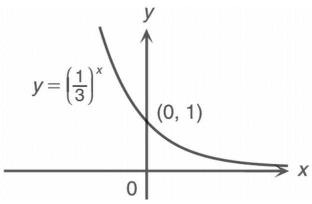


Classwork (p. 3.12)

(a)

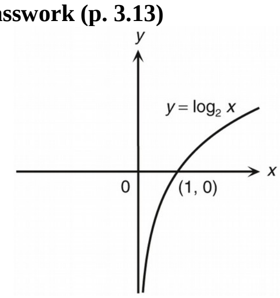


(b)

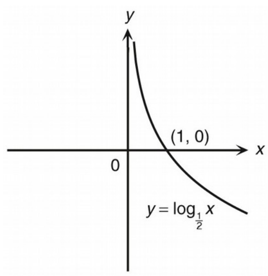


Classwork (p. 3.13)

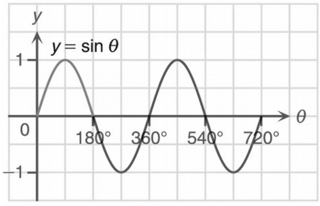
(a)

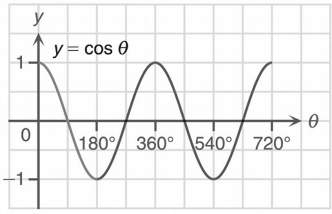


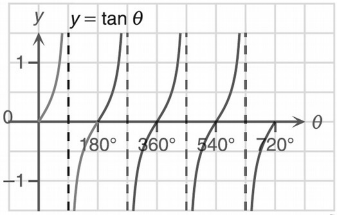
(b)



Classwork (p. 3.14)

(a) 

(b) 

(c) 

Classwork (p. 3.15)

1.

	Function		Maximum value	Minimum value
(a)	$y = \frac{x^2}{4} - 4$	:	no max. value	-4
(b)	$y = \sin x$	:	1	-1
(c)	$y = \cos x$	:	1	-1

2.

			Period
(a)	Graph $y = \sin x$	of :	360°
(b)	Graph $y = \cos x$	of :	360°

3. The graphs of $y = \frac{x^2}{4} - 4$ and $y = \cos x$ show reflectional symmetry about the *y*-axis.

4. The domain of the function $y = \log_{\frac{1}{10}} x$ is all positive real numbers, while the domain of other functions are all real numbers.

Classwork (p. 3.43)

(a) ∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = x^2 + 2x + 1$ upwards by 2 units.
∴ The algebraic representation of $g(x)$ is
$$g(x) = x^2 + 2x + 3.$$

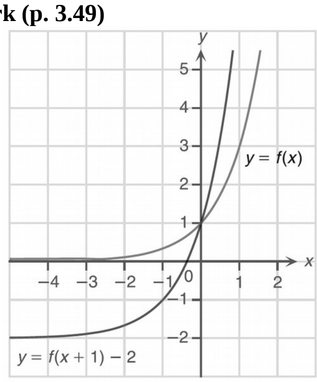
(b) ∵ The graph of $y = h(x)$ is obtained by translating the graph of $y = x^2 + 2x + 1$ downwards by 3 units.
∴ The algebraic representation of $h(x)$ is
$$h(x) = x^2 + 2x - 2.$$

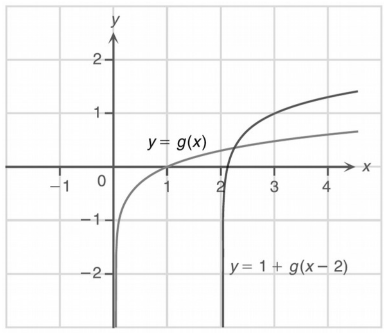
Classwork (p. 3.47)

(a) ∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = x^2 - x + 1$ rightwards by 2 units.
∴ The algebraic representation of $g(x)$ is
$$\begin{aligned} g(x) &= (x-2)^2 - (x-2) + 1 \\ &= x^2 - 4x + 4 - x + 2 + 1 \\ &= \underline{\underline{x^2 - 5x + 7}} \end{aligned}$$

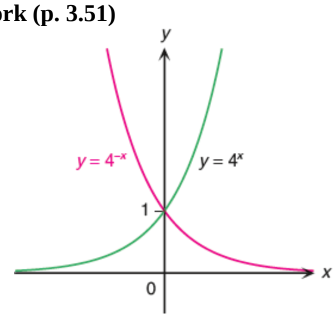
(b) ∵ The graph of $y = h(x)$ is obtained by translating the graph of $y = x^2 - x + 1$ leftwards by 3 units.
∴ The algebraic representation of $h(x)$ is
$$\begin{aligned} h(x) &= (x+3)^2 - (x+3) + 1 \\ &= x^2 + 6x + 9 - x - 3 + 1 \\ &= \underline{\underline{x^2 + 5x + 7}} \end{aligned}$$

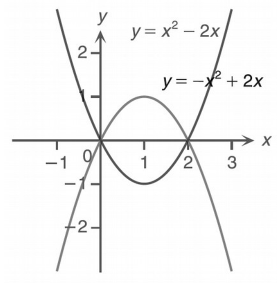
Classwork (p. 3.49)

1. (a) 

(b) 

Classwork (p. 3.51)

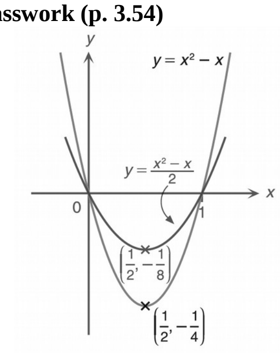
1. (a) 

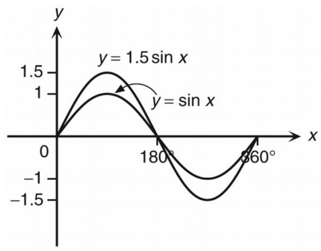
(b) 

2. **(a)** $g(x) = -(x^2 - 2x + 5)$
$= -x^2 + 2x - 5$

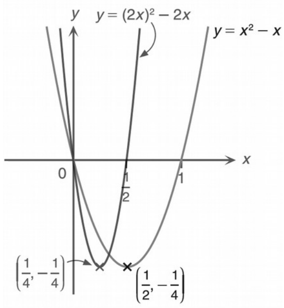
(b) $g(x) = (-x)^2 - 2(-x) + 5$
$= x^2 + 2x + 5$

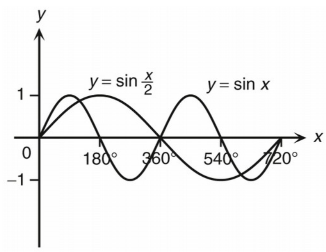
Classwork (p. 3.54)

(a) 

(b) 

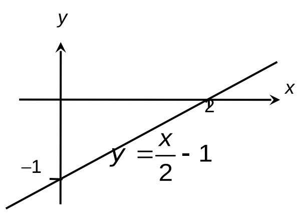
Classwork (p. 3.57)

(a) 

(b) 

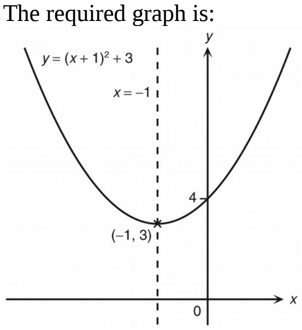
Quick Practice

Quick Practice 3.1 (p. 3.7)
The required graph is:

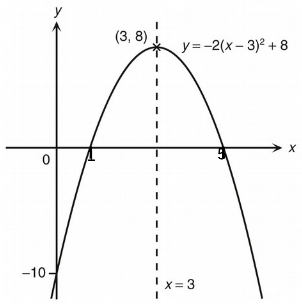


Quick Practice 3.2 (p. 3.10)

(a) The required graph is:

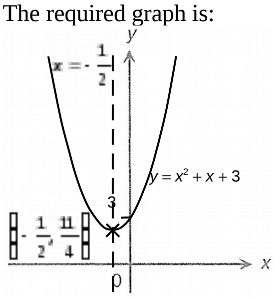


(b) The required graph is:



Quick Practice 3.3 (p. 3.11)

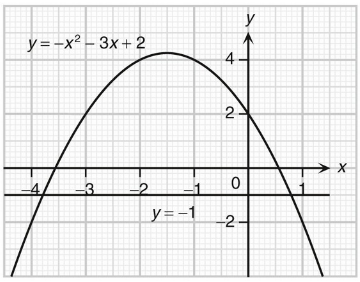
The required graph is:



Quick Practice 3.4 (p. 3.20)

Draw the horizontal line $y = -1$ on the graph of

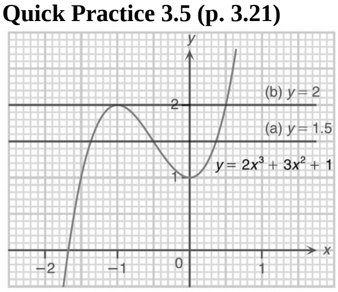
$y = -x^2 - 3x + 2$.



∵ The two graphs intersect at $x = -3.8$ and $x = 0.8$.

∴ The solutions of $-x^2 - 3x + 2 = -1$ are $x = -3.8$ or 0.8.

Quick Practice 3.5 (p. 3.21)



(a) Draw the horizontal line $y = 1.5$ on the graph of $y = 2x^3 + 3x^2 + 1$.

∵ The two graphs intersect at $x = -1.4$, $x = -0.5$ and $x = 0.4$.

∴ The solutions of $2x^3 + 3x^2 + 1 = 1.5$ are $x = -1.4$, -0.5 or 0.4.

(b) Draw the horizontal line $y = 2$ on the graph of $y = 2x^3 + 3x^2 + 1$.

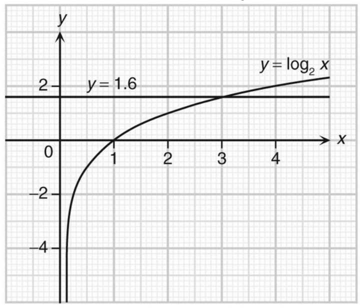
∵ The two graphs intersect at $x = -1.0$ and $x = 0.5$.

∴ The solutions of $2x^3 + 3x^2 + 1 = 1.5$ are $x = -1.0$ or 0.5.

Quick Practice 3.6 (p. 3.23)

(a)
$$5\log_2 x = 8$$
$$\log_2 x = 1.6$$

Draw the horizontal line $y = 1.6$ on the graph of $y = \log_2 x$.

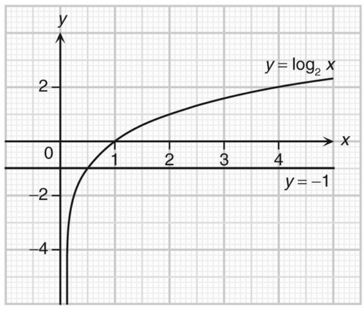


∵ The two graphs intersect at $x = 3.0$.

∴ The solution of $5\log_2 x = 8$ is $x = 3.0$.

(b)
$$2\log_2 x + 3 = 1$$
$$2\log_2 x = -2$$
$$\log_2 x = -1$$

Draw the horizontal line $y = -1$ on the graph of $y = \log_2 x$.

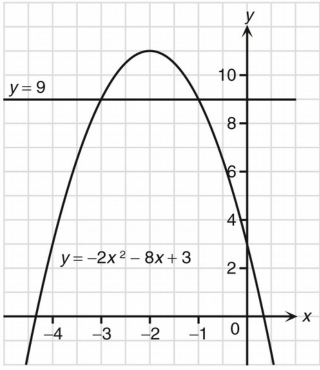


∵ The two graphs intersect at $x = 0.5$.

∴ The solution of $2\log_2 x + 3 = 1$ is $x = 0.5$.

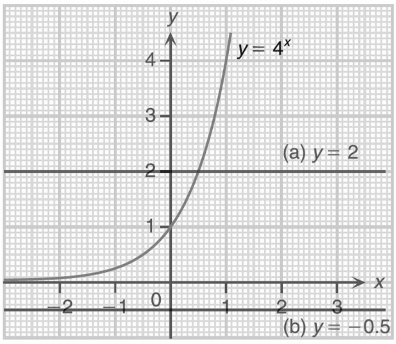
Quick Practice 3.7 (p. 3.30)

Draw the horizontal line $y = 9$ on the graph of $y = -2x^2 - 8x + 3$.



The two graphs intersect at $x = -3$ and $x = -1$.
For the range $-3 < x < -1$, the corresponding part of the graph of $y = -2x^2 - 8x + 3$ lies above the line $y = 9$.
∴ The solutions of $-2x^2 - 8x + 3 > 9$ are $-3 < x < -1$.

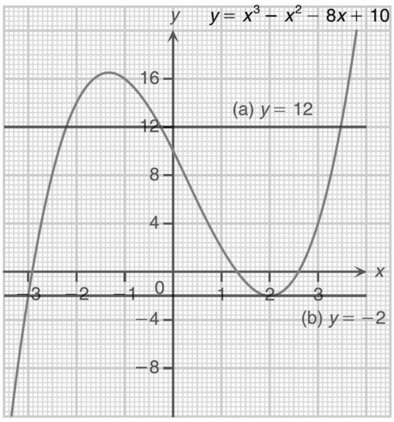
Quick Practice 3.8 (p. 3.31)



(a) Draw the horizontal line $y = 2$ on the graph of $y = 4^x$.
The two graphs intersect at $x = 0.5$.
For the range $x \geq 0.5$, the corresponding part of the graph of $y = 4^x$ lies on or above the line $y = 2$.
∴ The solutions of $4^x \geq 2$ are $x \geq 0.5$.

(b) Draw the horizontal line $y = -0.5$ on the graph of $y = 4^x$.
The two graphs do not intersect, and the whole graph of $y = 4^x$ lies above the line $y = -0.5$.
∴ The solutions of $4^x \geq -0.5$ are all real values of x.

Quick Practice 3.9 (p. 3.33)

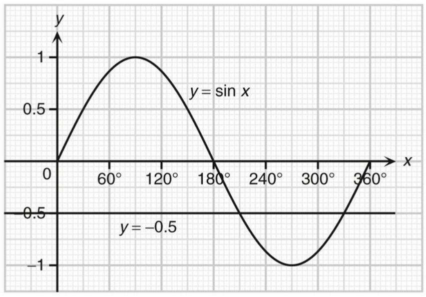


(a) Draw the horizontal line $y = 12$ on the graph of $y = x^3 - x^2 - 8x + 10$.
The two graphs intersect at $x = -2.2$, $x = -0.3$ and $x = 3.5$.
For the ranges $-2.2 \leq x \leq -0.3$ and $x \geq 3.5$, the corresponding parts of the graph of $y = x^3 - x^2 - 8x + 10$ lie on or above the line $y = 12$.
∴ The solutions of $x^3 - x^2 - 8x + 10 \geq 12$ are $-2.2 \leq x \leq -0.3$ or $x \geq 3.5$.

(b) Draw the horizontal line $y = -2$ on the graph of $y = x^3 - x^2 - 8x + 10$.
The two graphs intersect at $x = -3$ and $x = 2$.
For the range $x \geq -3$, the corresponding part of the graph of $y = x^3 - x^2 - 8x + 10$ lies on or above the line $y = -2$.
∴ The solutions of $x^3 - x^2 - 8x + 10 \geq -2$ are $x \geq -3$.

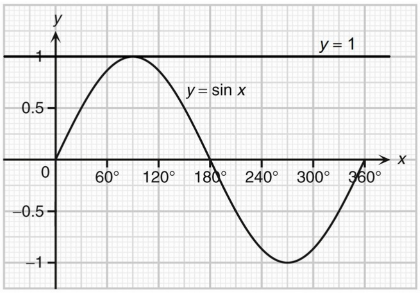
Quick Practice 3.10 (p. 3.35)

(a)
$$\begin{aligned} 2\sin x + 1 &< 0 \\ 2\sin x &< -1 \\ \sin x &< -0.5 \end{aligned}$$
Draw the horizontal line $y = -0.5$ on the graph of $y = \sin x$.



The two graphs intersect at $x = 210°$ and $x = 330°$.
For the range $210° < x < 330°$, the corresponding part of the graph of $y = \sin x$ lies below the line $y = -0.5$.
∴ For $0° \leq x \leq 360°$, the solutions of $2\sin x + 1 < 0$ are $210° < x < 330°$.

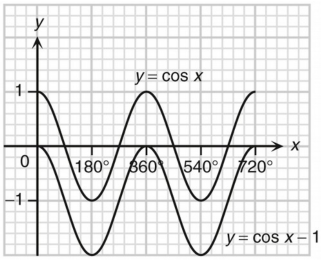
(b)
$$\begin{aligned} 2\sin x + 3 &< 5 \\ 2\sin x &< 2 \\ \sin x &< 1 \end{aligned}$$
Draw the horizontal line $y = 1$ on the graph of $y = \sin x$.



The two graphs intersect at $x = 90°$.
For the ranges $0° \leq x < 90°$ and $90° < x \leq 360°$, the corresponding parts of the graph of $y = \sin x$ lie below the line $y = 1$.
∴ For $0° \leq x \leq 360°$, the solutions of $2\sin x + 3 < 5$ are $0° \leq x < 90°$ or $90° < x \leq 360°$.

Quick Practice 3.11 (p. 3.44)

The graph of $y = \cos x - 1$ is obtained by translating the graph of $y = \cos x$ downwards by 1 unit.



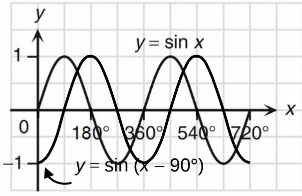
Quick Practice 3.12 (p. 3.45)

$$g(x) = x^2 + 3$$
$$= (x^2 - 8) + 11$$
$$= f(x) + 11$$

∴ The graph of $y = f(x)$ is translated upwards by 11 units.

Quick Practice 3.13 (p. 3.48)

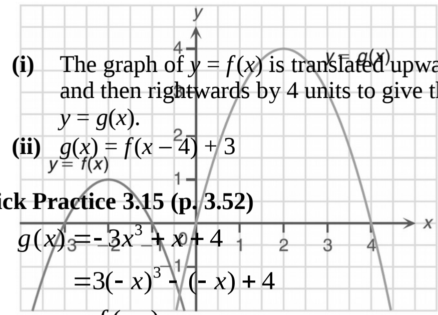
The graph of $y = \sin (x - 90°)$ is obtained by translating the graph of $y = \sin x$ rightwards by 90°.



Quick Practice 3.14 (p. 3.49)

(a) The vertices of the graphs of $y = f(x)$ and $y = g(x)$ are (–2, 1) and (2, 4) respectively.

(b)



(i) The graph of $y = f(x)$ is translated upwards by 3 units and then rightwards by 4 units to give the graph of $y = g(x)$.

(ii) $g(x) = f(x - 4) + 3$

Quick Practice 3.15 (p. 3.52)

(a)
$$g(x) = -3x^3 + x + 4$$
$$= 3(-x)^3 - (-x) + 4$$
$$= f(-x)$$

∴ The graph of $y = f(x)$ is reflected about the y-axis to give the graph of $y = g(x)$.

(b)
$$h(x) = -3x^3 + x - 4$$
$$= -(3x^3 - x + 4)$$
$$= -f(x)$$

∴ The graph of $y = f(x)$ is reflected about the x-axis to give the graph of $y = h(x)$.

Quick Practice 3.16 (p. 3.55)

(a) The graph of $y = f(x)$ is enlarged along the y-axis to 2 times the original to give the graph of $y = g(x)$.

(b)
$$g(x) = 2f(x)$$
$$= 2(2x^2 + 2x - 1)$$
$$= \underline{\underline{4x^2 + 4x - 2}}$$

Quick Practice 3.17 (p. 3.58)

(a)
$$g(x) = \log \frac{x}{4}$$
$$= f\left(\frac{x}{4}\right)$$

∴ The graph of $y = f(x)$ is enlarged along the x-axis to 4 times the original to give the graph of $y = g(x)$.

(b)
$$h(x) = \log 3x$$
$$= f(3x)$$

∴ The graph of $y = f(x)$ is reduced along the x-axis to $\frac{1}{3}$ times the original to give the graph of $y = h(x)$.

Quick Practice 3.18 (p. 3.60)

(a) Let the graph of $y = h(x)$ be the graph obtained by enlarging the graph of $y = f(x)$ along the x-axis to 2 times the original.

$\therefore$ $h(x) = f\left(\dfrac{x}{2}\right)$

$\because$ The graph of $y = g(x)$ is obtained by translating the graph of $y = h(x)$ downwards by 3 units.

$$g(x) = h(x) - 3$$
$$= f\left(\frac{x}{2}\right) - 3$$
$$\therefore \quad = \left[\left(\frac{x}{2} - 4\right)^2 + 6\right] - 3$$
$$= \frac{1}{4}(x - 8)^2 + 6 - 3$$
$$= \underline{\underline{\frac{1}{4}(x - 8)^2 + 3}}$$

(b) The coordinates of the vertex of the graph of $y = g(x)$ are (8, 3).

Quick Practice 3.19 (p. 3.61)

(a) From the graph,
y-intercept of the graph of $y = f(x)$ is 1,
y-intercept of the graph of $y = g(x)$ is –1

$$\therefore \quad k = 1 - (-1)$$
$$= \underline{\underline{2}}$$

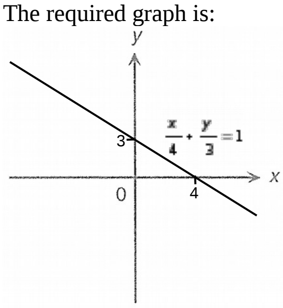
(b) The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ downwards by 2 units and then reflecting about the y-axis.

$$g(x) = f(-x) - 2$$
$$= \underline{\underline{2^{-x} - 2}}$$

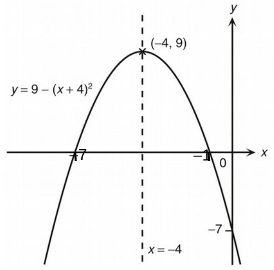
Further Practice

Further Practice (p. 3.11)

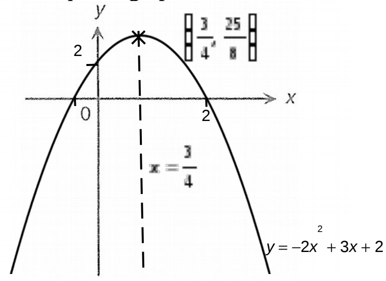
1. The required graph is:



2. The required graph is:



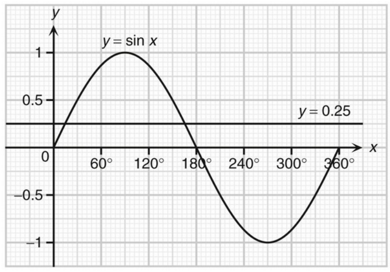
3. The required graph is:



Further Practice (p. 3.23)

1.
$$2\sin x = 0.5$$
$$\sin x = 0.25$$

Draw the horizontal line $y = 0.25$ on the graph of $y = \sin x$.

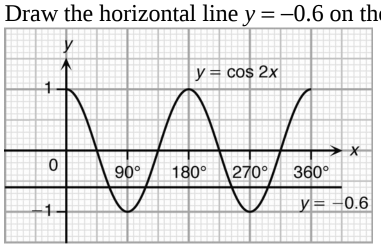


$\because$ The two graphs intersect at $x = 12°$ and $x = 168°$.

$\therefore$ For $0° \le x \le 360°$, the solutions of $2\sin x = 0.5$ are $x = 12°$ or $168°$.

2.
$$\cos 2x + 0.6 = 0$$
$$\cos 2x = -0.6$$

Draw the horizontal line $y = -0.6$ on the graph of $y = \cos 2x$.

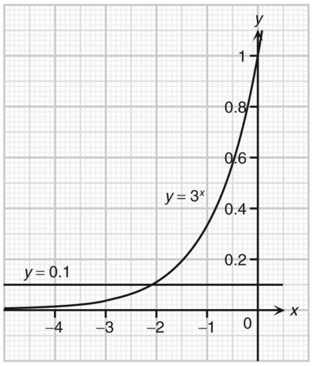


$\because$ The two graphs intersect at $x = 63°$, $x = 117°$, $x = 243°$ and $x = 297°$.

$\therefore$ For $0° \le x \le 360°$, the solutions of $\cos 2x + 0.6 = 0$ are $x = 63°$, $117°$, $243°$ or $297°$.

3.
$$3^{x+1} - 0.3 = 0$$
$$3(3^x) = 0.3$$
$$3^x = 0.1$$

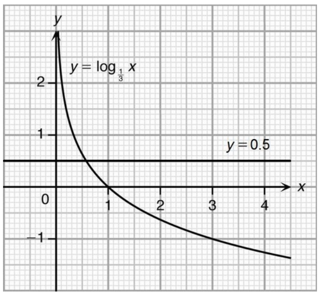
Draw the horizontal line $y = 0.1$ on the graph of $y = 3^x$.



∵ The two graphs intersect at $x = -2.1$.
∴ The solution of $3^{x+1} - 0.3 = 0$ is $x = -2.1$.

4.
$$3\log_{\frac{1}{3}} x^2 + 2 = 5$$
$$6\log_{\frac{1}{3}} x = 3$$
$$\log_{\frac{1}{3}} x = 0.5$$

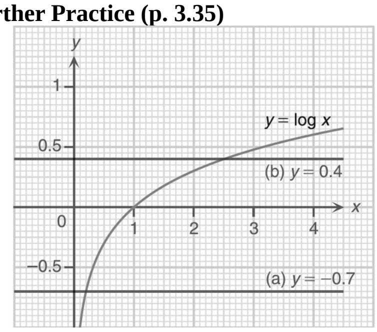
Draw the horizontal line $y = 0.5$ on the graph of $y = \log_{\frac{1}{3}} x$.



∵ The two graphs intersect at $x = 0.6$.
∴ The solution of $3\log_{\frac{1}{3}} x^2 + 2 = 5$ is $x = 0.6$.

Further Practice (p. 3.35)

1.



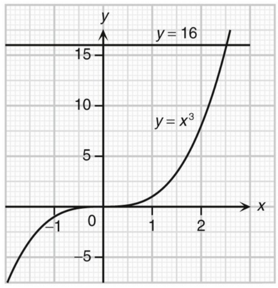
(a) Draw the horizontal line $y = -0.7$ on the graph of $y = \log x$.
The two graphs intersect at $x = 0.2$.
For the range $x \ge 0.2$, the corresponding part of the graph of $y = \log x$ lies on or above the line $y = -0.7$.
∴ The solutions of $\log x \ge -0.7$ are $x \ge 0.2$.

(b)
$$-\log x \ge -0.4$$
$$\log x \le 0.4$$

Draw the horizontal line $y = 0.4$ on the graph of $y = \log x$.
The two graphs intersect at $x = 2.5$.
For the range $0 < x \le 2.5$, the corresponding part of the graph of $y = \log x$ lies on or below the line $y = 0.4$.
∴ The solutions of $-\log x \ge -0.4$ are $0 < x \le 2.5$.

2.
$$\frac{1}{2}x^3 - 5 \ge 3$$
$$\frac{1}{2}x^3 \ge 8$$
$$x^3 \ge 16$$

Draw the horizontal line $y = 16$ on the graph of $y = x^3$.



The two graphs intersect at $x = 2.5$.
For the range $x \ge 2.5$, the corresponding part of the graph of $y = x^3$ lies on or above the line $y = 16$.
∴ The solutions of $\frac{1}{2}x^3 - 5 \ge 3$ are $x \ge 2.5$.

Further Practice (p. 3.62)

1. (a) (iii)
The graph of $y = \cos x + 1$ is obtained by translating the graph of $y = \cos x$ upwards by 1 unit.

(b) (i)
The graph of $y = 2\cos x$ is obtained by enlarging the graph of $y = \cos x$ along the y-axis to 2 times the original.

(c) (ii)
The graph of $y = \cos\frac{x}{2}$ is obtained by enlarging the graph of $y = \cos x$ along the x-axis to 2 times the original.

2. (a)
$$g(x) = x^2 - 2x + 3$$
$$= (x-1)^2 + 2$$
$$= f(x) + 2$$

∴ The graph of $y = f(x)$ is translated upwards by 2 units to give the graph of $y = g(x)$.

(b)
$$g(x) = x^2 + 2x + 1$$
$$= (x+1)^2$$
$$= (x+2-1)^2$$
$$= f(x+2)$$

∴ The graph of $y = f(x)$ is translated leftwards by 2 units to give the graph of $y = g(x)$.

Alternative Solution

$$g(x) = x^2 + 2x + 1$$
$$= (x+1)^2$$
$$= (-x-1)^2$$
$$= f(-x)$$

∴ The graph of $y = f(x)$ is reflected about the y-axis to give the graph of $y = g(x)$.

3. **(a)** Let the graph of $y = h(x)$ be the graph obtained by reflecting the graph of $y = f(x)$ about the x-axis.

∴ $h(x) = -f(x)$

The graph of $y = g(x)$ is obtained by translating the graph of $y = h(x)$ leftwards by 2 units.

$$g(x) = h(x+2)$$
$$\therefore \quad = -f(x+2)$$
$$= -\{[(x+2)+2]^2 - 5\}$$
$$= \underline{\underline{5-(x+4)^2}}$$

(b) The coordinates of the vertex of the graph of $y = g(x)$ are (–4, 5).

Exercise

Exercise 3A (p. 3.16)
Level 1

1. The domain of the function $y = -2x + 1$ is all real numbers. The graph of the function has no axis of symmetry. The function has neither maximum value nor minimum value.

2. The domain of the function $y = -x^2 - 4x + 5$ is all real numbers.

$$y = -x^2 - 4x + 5$$
$$= -(x^2 + 4x) + 5$$
$$= -(x^2 + 4x + 4) + 4 + 5$$
$$= -(x+2)^2 + 9$$

∴ The axis of symmetry is $x = -2$.
The maximum value of the function is 9.

3. The domain of the function $y = \left(\frac{1}{2}\right)^x$ is all real numbers.

The graph of the function has no axis of symmetry. The function has neither maximum value nor minimum value.

4. The domain of the function $y = 2\log x$ is all positive real numbers.
The graph of the function has no axis of symmetry. The function has neither maximum value nor minimum value.

5. The domain of the function $y = \cos x$ is all real numbers. The period of the function is 360°. The maximum value and the minimum value of the function are 1 and –1 respectively.

6. The domain of the function $y = \sin 2x$ is all real numbers. The period of the function is 180°. The maximum value and the minimum value of the function are 1 and –1 respectively.

7. The domain of the function $y = -\tan x$ is all real numbers except $\pm 90^\circ$, $\pm 270^\circ$, etc.
The period of the function is 180°. The function has neither maximum value nor minimum value.

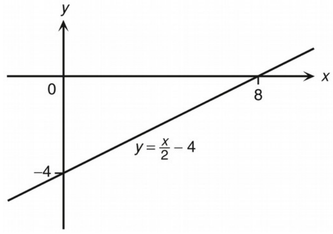
8. **(a)** When $x = 0$, $y = \frac{0}{2} - 4 = -4$

∴ The y-intercept is –4.

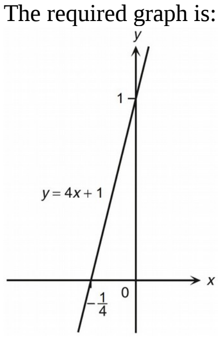
When $y = 0$,
$$0 = \frac{x}{2} - 4$$
$$4 = \frac{x}{2}$$
$$x = 8$$

∴ The x-intercept is 8.

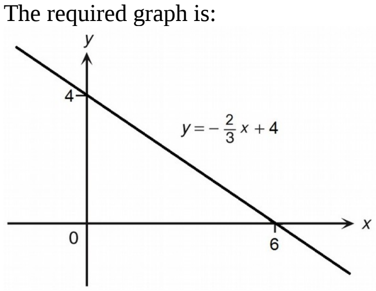
(b) The required graph is:



9. **(a)** The required graph is:



(b) The required graph is:



10. **(a)** **(i)** Its axis of symmetry: $x = 1$

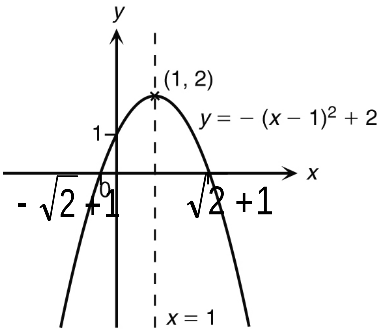
(ii) The coordinates of the vertex: (1, 2)

(iii) ∵ Coefficient of $x^2 = -1 < 0$
∴ The graph opens downwards.

(iv) When $x = 0$, $y = -(0-1)^2 + 2 = 1$
∴ Its y-intercept is 1.

(b) When $y = 0$,
$$0 = -(x-1)^2 + 2$$
$$(x-1)^2 = 2$$
$$x - 1 = \pm\sqrt{2}$$
$$x = -\sqrt{2} + 1 \text{ or } x = \sqrt{2} + 1$$

∴ Its x-intercepts are $-\sqrt{2} + 1$ and $\sqrt{2} + 1$.
The required graph is:



11. **(a)**

$$y = x^2 + 4x + 3$$
$$= [x^2 + 2(2)(x) + 4] - 1$$
$$= (x + 2)^2 - 1$$

(i) Its axis of symmetry: $x = -2$

(ii) The coordinates of the vertex: $(-2, -1)$

(iii) ∵ Coefficient of $x^2 = 1 > 0$

∴ The graph opens upwards.

(iv) When $x = 0$, $y = 0^2 + 4(0) + 3 = 3$

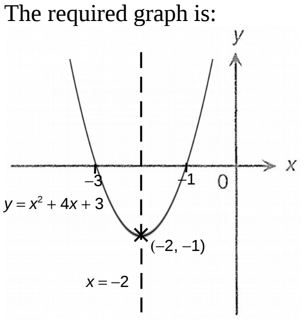
∴ Its y-intercept is 3.

(b) When $y = 0$,

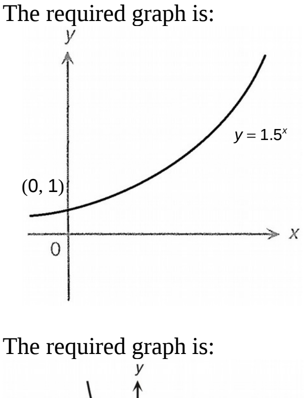
$$0 = (x + 2)^2 - 1$$
$$(x + 2)^2 = 1$$
$$x + 2 = \pm 1$$
$$x = -3 \text{ or } x = -1$$

∴ Its x-intercepts are −3 and −1.

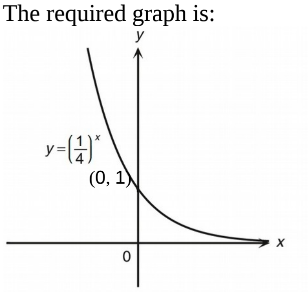
The required graph is:



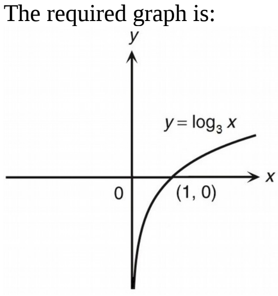
12. **(a)** The required graph is:



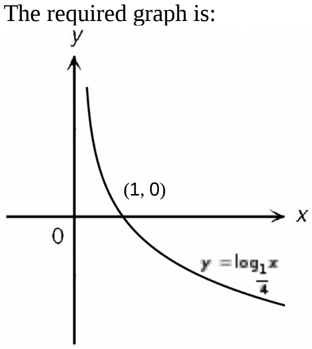
(b) The required graph is:



13. **(a)** The required graph is:

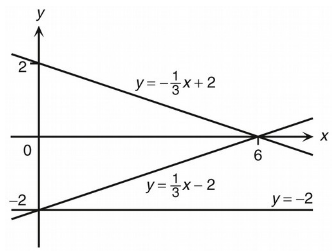


(b) The required graph is:



Level 2

14. **(a), (b)**

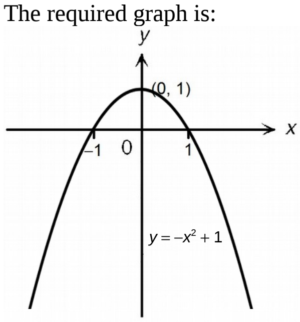


15. **(a)** When $y = 0$,

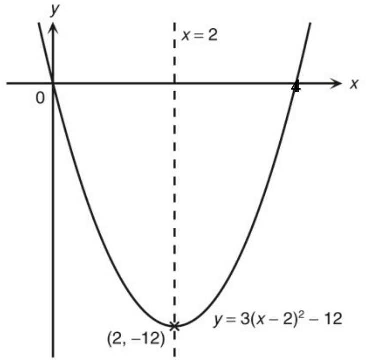
$$0 = -x^2 + 1$$
$$x^2 = 1$$
$$x = \pm 1$$

∴ The x-intercepts are −1 and 1.

(b) The required graph is:

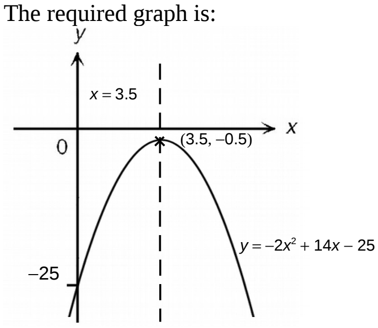


16. **(a)** The required graph is:



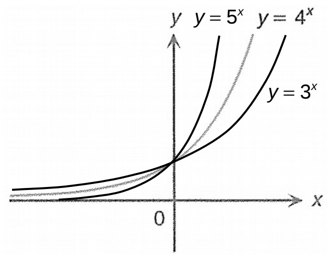
(b) The minimum value of the function is −12.

17. (a) The required graph is:

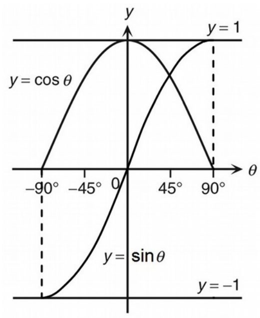


(b) The maximum value of the function is −0.5.

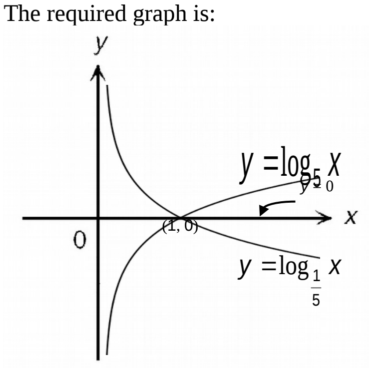
18. (a), (b)



19. (a), (b)



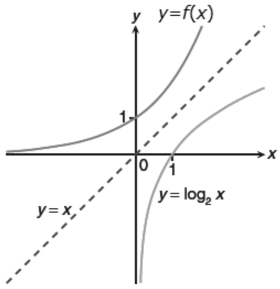
20. (a) The required graph is:



(b) Yes, the two graphs in (a) show reflectional symmetry with each other about the *x*-axis.

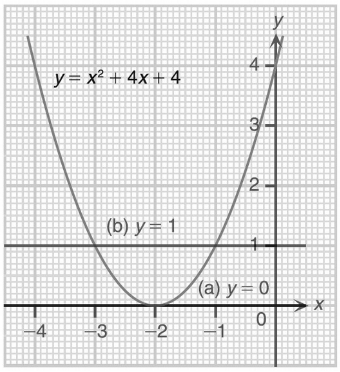
21. (a) $y = 2^x$

(b)



Exercise 3B (p. 3.24)
Level 1

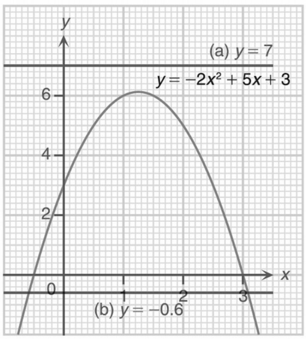
1.



(a) Draw the horizontal line $y = 0$ on the graph of $y = x^2 + 4x + 4$.
∵ The two graphs intersect at $x = -2.0$.
∴ The solution of $x^2 + 4x + 4 = 0$ is $x = -2.0$.

(b) Draw the horizontal line $y = 1$ on the graph of $y = x^2 + 4x + 4$.
∵ The two graphs intersect at $x = -3.0$ and $x = -1.0$.
∴ The solutions of $x^2 + 4x + 4 = 1$ are $x = -3.0$ or -1.0.

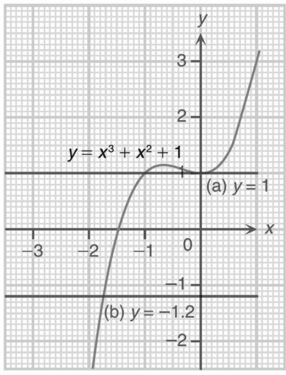
2.



(a) Draw the horizontal line $y = 7$ on the graph of $y = -2x^2 + 5x + 3$.
∵ The two graphs do not intersect.
∴ The equation $-2x^2 + 5x + 3 = 7$ has no real solutions.

(b) Draw the horizontal line $y = -0.6$ on the graph of $y = -2x^2 + 5x + 3$.
∵ The two graphs intersect at $x = -0.6$ and $x = 3.1$.
∴ The solutions of $-2x^2 + 5x + 3 = -0.6$ are $x = -0.6$ or 3.1.

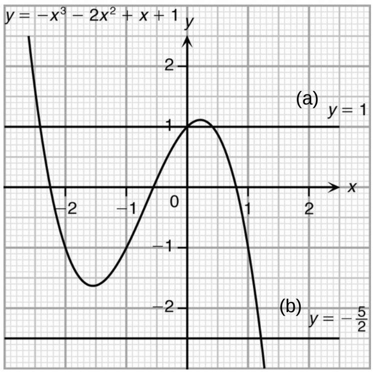
3.



(a) Draw the horizontal line $y = 1$ on the graph of $y = x^3 + x^2 + 1$.
∵ The two graphs intersect at $x = -1.0$ and $x = 0.0$.
∴ The solutions of $x^3 + x^2 + 1 = 1$ are $x = -1.0$ or 0.0.

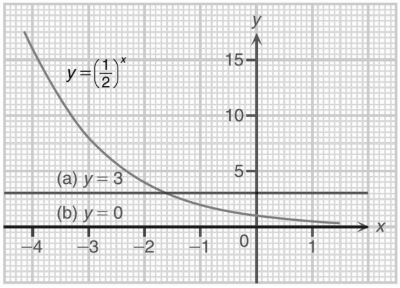
(b) Draw the horizontal line $y = -1.2$ on the graph of $y = x^3 + x^2 + 1$.
∵ The two graphs intersect at $x = -1.7$.
∴ The solution of $x^3 + x^2 + 1 = -1.2$ is $x = -1.7$.

4.



(a) Draw the horizontal line $y = 1$ on the graph of $y = -x^3 - 2x^2 + x + 1$.
∵ The two graphs intersect at $x = -2.4$, $x = 0.0$ and $x = 0.4$.
∴ The solutions of $-x^3 - 2x^2 + x + 1 = 1$ are $x = -2.4$, 0.0 or 0.4.

(b) Draw the horizontal line $y = -\frac{5}{2}$ on the graph of $y = -x^3 - 2x^2 + x + 1$.
∵ The two graphs intersect at $x = 1.2$.
∴ The solution of $-x^3 - 2x^2 + x + 1 = -\frac{5}{2}$ is $x = 1.2$.

5.


(a) Draw the horizontal line $y = 3$ on the graph of

$$y = \left(\frac{1}{2}\right)^x.$$

∵ The two graphs intersect at $x = -1.6$.

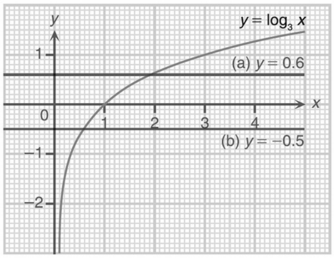
∴ The solution of $\left(\frac{1}{2}\right)^x = 3$ is $x = -1.6$.

(b) Draw the horizontal line $y = 0$ on the graph of

$$y = \left(\frac{1}{2}\right)^x.$$

∵ The two graphs do not intersect.

∴ $\left(\frac{1}{2}\right)^x = 0$ has no real solutions.

6.


(a) Draw the horizontal line $y = 0.6$ on the graph of $y = \log_3 x$.

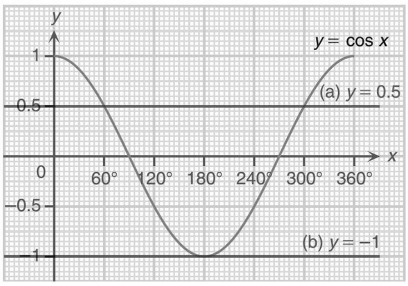
∵ The two graphs intersect at $x = 1.9$.

∴ The solution of $\log_3 x = 0.6$ is $x = 1.9$.

(b) Draw the horizontal line $y = -0.5$ on the graph of $y = \log_3 x$.

∵ The two graphs intersect at $x = 0.6$.

∴ The solution of $\log_3 x = -0.5$ is $x = 0.6$.

7.


(a) Draw the horizontal line $y = 0.5$ on the graph of $y = \cos x$.

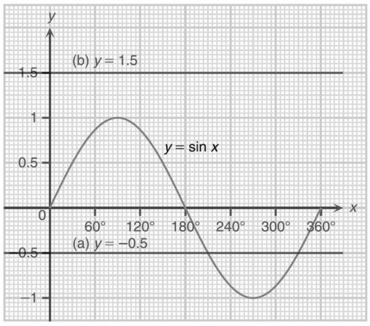
∵ The two graphs intersect at $x = 60°$ and $x = 300°$.

∴ The solutions of $\cos x = 0.5$ are $x = 60°$ or $300°$.

(b) Draw the horizontal line $y = -1$ on the graph of $y = \cos x$.

∵ The two graphs intersect at $x = 180°$.

∴ The solution of $\cos x = -1$ is $x = 180°$.

8.


(a)
$$\sin x + 0.5 = 0$$
$$\sin x = -0.5$$

Draw the horizontal line $y = -0.5$ on the graph of $y = \sin x$.

∵ The two graphs intersect at $x = 210°$ and $x = 330°$.

∴ For $0° \le x \le 360°$, the solutions of $\sin x + 0.5 = 0$ are $x = 210°$ or $330°$.

(b)
$$\sin x - 1.5 = 0$$
$$\sin x = 1.5$$

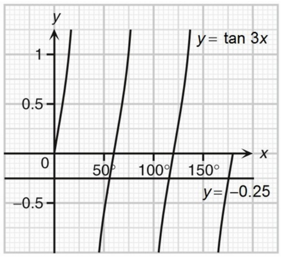
Draw the horizontal line $y = 1.5$ on the graph of $y = \sin x$.

∵ The two graphs do not intersect.

∴ For $0° \le x \le 360°$, $\sin x - 1.5 = 0$ has no real solutions.

9. **(a)**
$$4\tan 3x + 1 = 0$$
$$4\tan 3x = -1$$
$$\tan 3x = -0.25$$

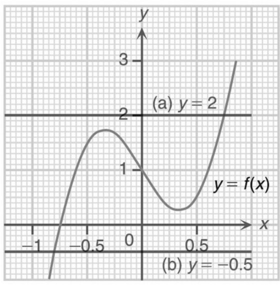
(b) Draw the horizontal line $y = -0.25$ on the graph of $y = \tan 3x$.



∵ The two graphs intersect at $x = 55°$, $x = 115°$ and $x = 175°$.

∴ For $0° \le x \le 180°$, the solutions of $4 \tan 3x + 1 = 0$ are $x = 55°$, $115°$ or $175°$.

Level 2

10.


(a)
$$3f(x) = 6$$
$$f(x) = 2$$

Draw the horizontal line $y = 2$ on the graph of $y = f(x)$.

∵ The two graphs intersect at $x = 0.75$.

∴ The solution of $3f(x) = 6$ is $x = 0.75$.

(b)
$$2f(x) + 1 = 0$$
$$2f(x) = -1$$
$$f(x) = -0.5$$

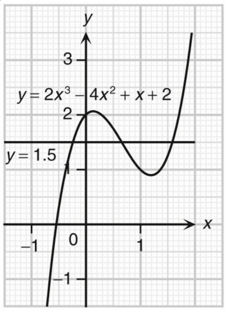
Draw the horizontal line $y = -0.5$ on the graph of $y = f(x)$.

∵ The two graphs intersect at $x = -0.80$.

∴ The solution of $2f(x) + 1 = 0$ is $x = -0.80$.

11.
$$4x^3 - 8x^2 + 2x + 4 = 3$$
$$2(2x^3 - 4x^2 + x + 2) = 3$$
$$2x^3 - 4x^2 + x + 2 = 1.5$$

Draw the horizontal line $y = 1.5$ on the graph of $y = 2x^3 - 4x^2 + x + 2$.

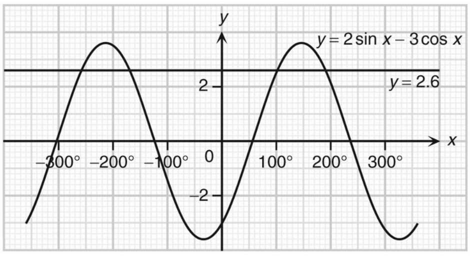


∵ The two graphs intersect at $x = -0.2$, $x = 0.7$ and $x = 1.6$.

∴ The solutions of $4x^3 - 8x^2 + 2x + 4 = 3$ are $x = -0.2$, 0.7 or 1.6.

12.
$$10\sin x - 15\cos x = 13$$
$$5(2\sin x - 3\cos x) = 13$$
$$2\sin x - 3\cos x = 2.6$$

Draw the horizontal line $y = 2.6$ on the graph of $y = 2\sin x - 3\cos x$.

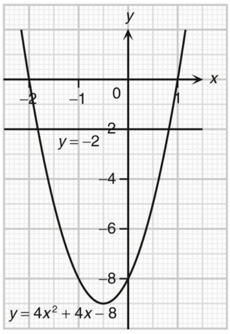


∵ The two graphs intersect at $x = -260°$, $x = -170°$, $x = 100°$ and $x = 190°$.

∴ For $-360° \le x \le 360°$, the solutions of $10\sin x - 15\cos x = 13$ are $x = -260°$, $-170°$, $100°$ or $190°$.

13.
$$4x^2 + 4x - 6 = 0$$
$$4x^2 + 4x - 8 + 2 = 0$$
$$4x^2 + 4x - 8 = -2$$

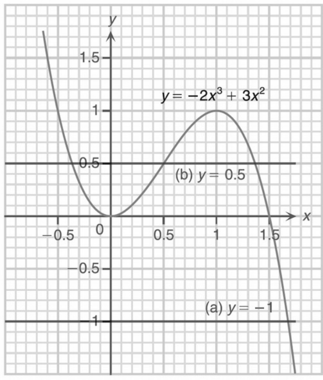
Draw the horizontal line $y = -2$ on the graph of $y = 4x^2 + 4x - 8$.



∵ The two graphs intersect at $x = -1.8$ and $x = 0.8$.

∴ The solutions of $4x^2 + 4x - 6 = 0$ are $x = -1.8$ or 0.8.

14.



(a)
$$2x^3 - 3x^2 - 1 = 0$$
$$-2x^3 + 3x^2 = -1$$

Draw the horizontal line $y = -1$ on the graph of $y = -2x^3 + 3x^2$.

∵ The two graphs intersect at $x = 1.7$.

∴ The solution of $2x^3 - 3x^2 - 1 = 0$ is $x = 1.7$.

(b)
$$4x^3 - 6x^2 + 1 = 0$$
$$2(-2x^3 + 3x^2) = 1$$
$$-2x^3 + 3x^2 = 0.5$$

Draw the horizontal line $y = 0.5$ on the graph of $y = -2x^3 + 3x^2$.

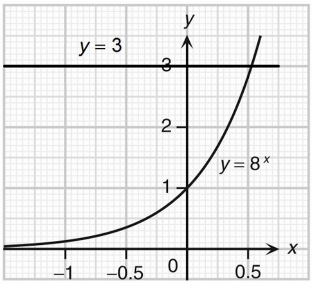
∵ The two graphs intersect at $x = -0.4$, $x = 0.5$ and $x = 1.4$.

∴ The solutions of $4x^3 - 6x^2 + 1 = 0$ are $x = -0.4$, 0.5 or 1.4.

15. (a)
$$2^{3x} = (2^3)^x$$
$$= 8^x$$

(b)
$$2^{3x+2} = 12$$
$$2^{3x} \cdot 2^2 = 12$$
$$4 \cdot 8^x = 12 \text{ (from (a))}$$
$$8^x = 3$$

Draw the horizontal line $y = 3$ on the graph of $y = 8^x$.

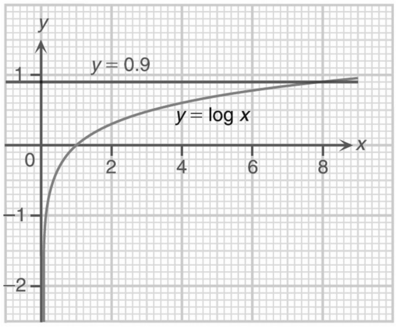


∵ The two graphs intersect at $x = 0.55$.
∴ The solution of $2^{3x+2} = 12$ is $x = 0.55$.

16. (a)
$$\log\left(\frac{x^5}{100}\right) = \log x^5 - \log 100$$
$$= 5\log x - 2$$

(b)
$$\log\left(\frac{x^5}{100}\right) = 2.5$$
$$5\log x - 2 = 2.5 \text{ (from (a))}$$
$$5\log x = 4.5$$
$$\log x = 0.9$$

Draw the horizontal line $y = 0.9$ on the graph of $y = \log x$.



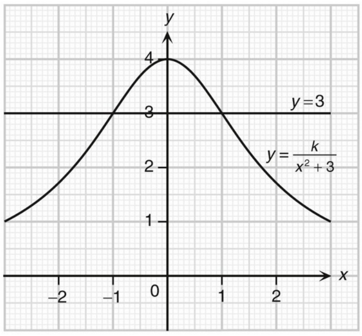
∵ The two graphs intersect at $x = 8.0$.
∴ The solution of $\log\left(\frac{x^5}{100}\right) = 2.5$ is $x = 8.0$.

17. (a) The graph of $y = \frac{k}{x^2+3}$ passes through (0, 4).

By substituting (0, 4) into $y = \frac{k}{x^2+3}$, we have
$$4 = \frac{k}{(0)^2+3}$$
$$k = \underline{\underline{12}}$$

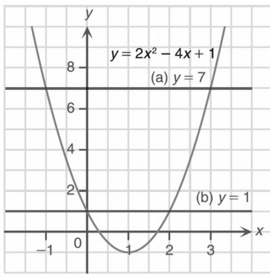
(b)
$$\frac{24}{x^2+3} = 6$$
$$\frac{12}{x^2+3} = 3$$

Draw the horizontal line $y = 3$ on the graph of $y = \frac{12}{x^2+3}$.



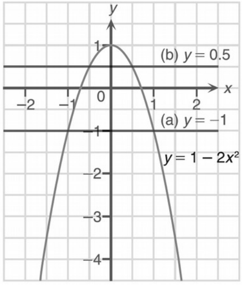
∵ The two graphs intersect at $x = -1.0$ and $x = 1.0$.
∴ The solutions of $\frac{24}{x^2+3} = 6$ are $x = -1.0$ or 1.0.

Exercise 3C (p. 3.36)
Level 1

1.


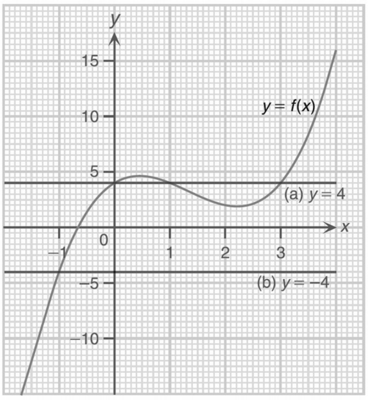
(a) Draw the horizontal line $y = 7$ on the graph of $y = 2x^2 - 4x + 1$.
The two graphs intersect at $x = -1$ and $x = 3$.
For the ranges $x < -1$ and $x > 3$, the corresponding parts of the graph of $y = 2x^2 - 4x + 1$ lie above the line $y = 7$.
∴ The solutions of $2x^2 - 4x + 1 > 7$ are $x < -1$ or $x > 3$.

(b) Draw the horizontal line $y = 1$ on the graph of $y = 2x^2 - 4x + 1$.
The two graphs intersect at $x = 0$ and $x = 2$.
For the range $0 < x < 2$, the corresponding part of the graph of $y = 2x^2 - 4x + 1$ lies below the line $y = 1$.
∴ The solutions of $2x^2 - 4x + 1 < 1$ are $0 < x < 2$.

2. 

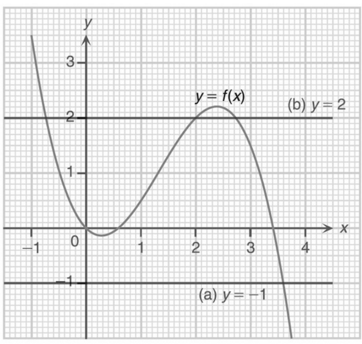
(a) Draw the horizontal line $y = -1$ on the graph of $y = 1 - 2x^2$.
The two graphs intersect at $x = -1$ and $x = 1$.
For the ranges $x \le -1$ or $x \ge 1$, the corresponding parts of the graph of $y = 1 - 2x^2$ lie on or below the line $y = -1$.
∴ The solutions of $1 - 2x^2 \le -1$ are $x \le -1$ or $x \ge 1$.

(b) Draw the horizontal line $y = 0.5$ on the graph of $y = 1 - 2x^2$.
The two graphs intersect at $x = -0.5$ and $x = 0.5$.
For the range $-0.5 \le x \le 0.5$, the corresponding part of the graph of $y = 1 - 2x^2$ lies on or above the line $y = 0.5$.
∴ The solutions of $1 - 2x^2 \ge 0.5$ are $-0.5 \le x \le 0.5$.

3. 

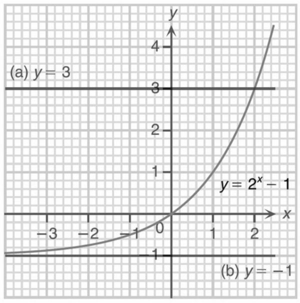
(a) Draw the horizontal line $y = 4$ on the graph of $y = f(x)$.
The two graphs intersect at $x = 0$, $x = 1$ and $x = 3$.
For the ranges $x \le 0$ and $1 \le x \le 3$, the corresponding parts of the graph of $y = f(x)$ lie on or below the line $y = 4$.
∴ The solutions of $f(x) \le 4$ are $x \le 0$ or $1 \le x \le 3$.

(b) Draw the horizontal line $y = -4$ on the graph of $y = f(x)$.
The two graphs intersect at $x = -1$.
For the range $x \ge -1$, the corresponding part of the graph of $y = f(x)$ lies on or above the line $y = -4$.
∴ The solutions of $f(x) \ge -4$ are $x \ge -1$.

4. 

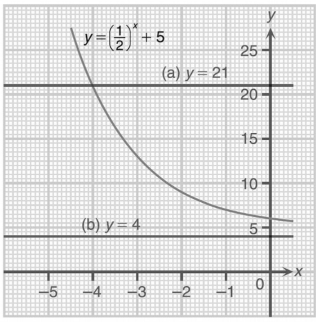
(a) Draw the horizontal line $y = -1$ on the graph of $y = f(x)$.
The two graphs intersect at $x = 3.6$.
For the range $x > 3.6$, the corresponding part of the graph of $y = f(x)$ lies below the line $y = -1$.
∴ The solutions of $f(x) < -1$ are $x > 3.6$.

(b) Draw the horizontal line $y = 2$ on the graph of $y = f(x)$.
The two graphs intersect at $x = -0.7$, $x = 2$ and $x = 2.7$.
For the ranges $x < -0.7$ and $2 < x < 2.7$, the corresponding parts of the graph of $y = f(x)$ lie above the line $y = 2$.
∴ The solutions of $f(x) > 2$ are $x < -0.7$ or $2 < x < 2.7$.

5. 

(a) Draw the horizontal line $y = 3$ on the graph of $y = 2^x - 1$.
The two graphs intersect at $x = 2$.
For the range $x < 2$, the corresponding part of the graph of $y = 2^x - 1$ lies below the line $y = 3$.
∴ The solutions of $2^x - 1 < 3$ are $x < 2$.

(b) Draw the horizontal line $y = -1$ on the graph of $y = 2^x - 1$.
The two graphs do not intersect, and the whole graph of $y = 2^x - 1$ lies above the line $y = -1$.
∴ The inequality $2^x - 1 < -1$ has no real solutions.

6. 

(a) Draw the horizontal line $y = 21$ on the graph of

$$y = \left(\frac{1}{2}\right)^x + 5.$$

The two graphs intersect at $x = -4$.
For the range $x \le -4$, the corresponding part of the graph of $y = \left(\frac{1}{2}\right)^x + 5$ lies on or above the line $y = 21$.
∴ The solutions of $\left(\frac{1}{2}\right)^x + 5 \ge 21$ are $x \le -4$.

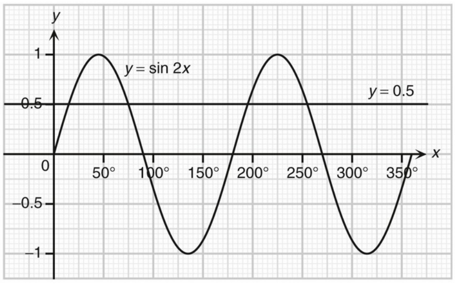
(b) Draw the horizontal line $y = 4$ on the graph of

$y = \left(\frac{1}{2}\right)^x + 5$.

The two graphs do not intersect, and the whole graph of $y = \left(\frac{1}{2}\right)^x + 5$ lies above the line $y = 4$.

∴ The solutions of $\left(\frac{1}{2}\right)^x + 5 \geq 4$ are all real values of x.

7. Draw the horizontal line $y = 0.5$ on the graph of $y = \sin 2x$.

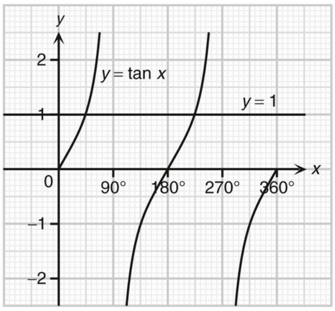


The two graphs intersect at $x = 15°$, $x = 75°$, $x = 195°$ and $x = 255°$.
For the ranges $15° < x < 75°$ and $195° < x < 255°$, the corresponding parts of the graph of $y = \sin 2x$ lie above the line $y = 0.5$.
∴ For $0° \leq x \leq 360°$, the solutions of $\sin 2x > 0.5$ are $15° < x < 75°$ or $195° < x < 255°$.

8. $\tan x - 1 < 0$

$\tan x < 1$

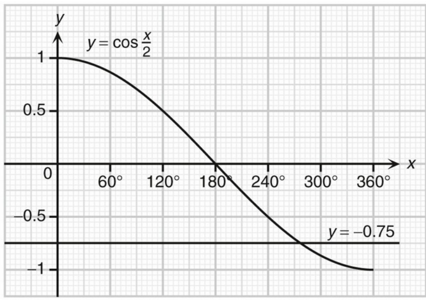
Draw the horizontal line $y = 1$ on the graph of $y = \tan x$.



The two graphs intersect at $x = 45°$ and $x = 225°$.
For the ranges $0° \leq x < 45°$, $90° < x < 225°$ and $270° < x \leq 360°$, the corresponding parts of the graph of $y = \tan x$ lie below the line $y = 1$.
∴ For $0° \leq x \leq 360°$, the solutions of $\tan x - 1 < 0$ are $0° \leq x < 45°$, $90° < x < 225°$ or $270° < x \leq 360°$.

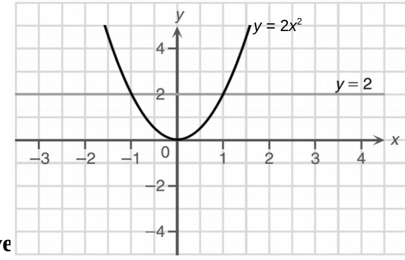
9. **(a)**
$$4\cos\frac{x}{2} + 3 \leq 0$$
$$4\cos\frac{x}{2} \leq -3$$
$$\cos\frac{x}{2} \leq -0.75$$

(b) Draw the horizontal line $y = -0.75$ on the graph of $y = \cos\frac{x}{2}$.



The two graphs intersect at $x = 276°$.
For the range $276° \leq x \leq 360°$, the corresponding part of the graph of $y = \cos\frac{x}{2}$ lies on or below the line $y = -0.75$.
∴ For $0° \leq x \leq 360°$, the solutions of $4\cos\frac{x}{2} + 3 \leq 0$ are $276° \leq x \leq 360°$.

10.



ıt $x = -1$ and

1.

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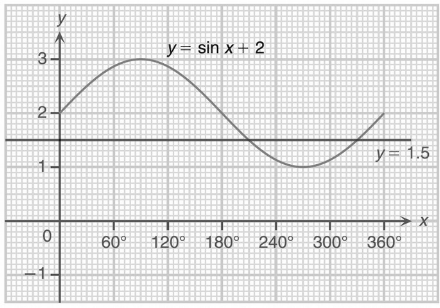
Leve

11. $\sin x \leq -0.5$

$\sin x + 2 \leq 1.5$

Draw the horizontal line $y = 1.5$ on the graph of $y = \sin x + 2$.



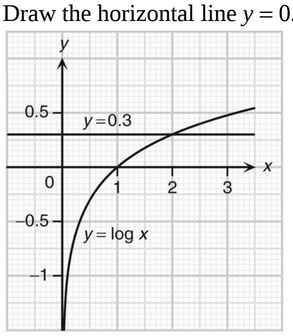
The two graphs intersect at $x = 210°$ and $x = 330°$.
For the range $210° \leq x \leq 330°$, the corresponding part of the graph of $y = \sin x + 2$ lies on or below the line $y = 1.5$.
∴ For $0° \leq x \leq 360°$, the solutions of $\sin x \leq -0.5$ are $210° \leq x \leq 330°$.

12. $10\log x - 3 \leq 0$

$10\log x \leq 3$

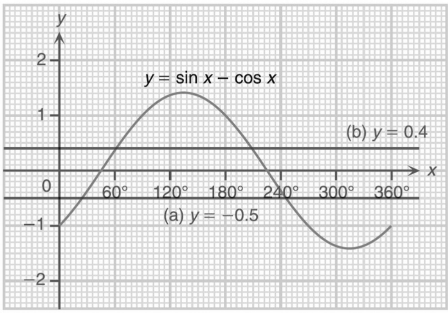
$\log x \leq 0.3$

Draw the horizontal line $y = 0.3$ on the graph of $y = \log x$.



The two graphs intersect at $x = 2$.
For the range $0 < x \le 2$, the corresponding part of the graph of $y = \log x$ lies on or below the line $y = 0.3$.
$\therefore$ The solutions of $10 \log x - 3 \le 0$ are $0 < x \le 2$.

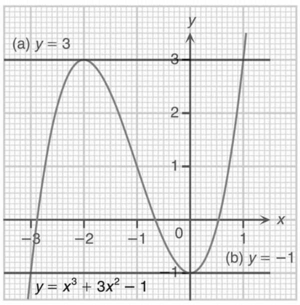
13.



(a)
$$2\sin x - 2\cos x + 3 > 2$$
$$2(\sin x - \cos x) > -1$$
$$\sin x - \cos x > -0.5$$
Draw the horizontal line $y = -0.5$ on the graph of $y = \sin x - \cos x$.
The two graphs intersect at $x = 24°$ and $x = 246°$.
For the range $24° < x < 246°$, the corresponding part of the graph of $y = \sin x - \cos x$ lies above the line $y = -0.5$.
$\therefore$ For $0° \le x \le 360°$, the solutions of $2\sin x - 2\cos x + 3 > 2$ are $24° < x < 246°$.

(b)
$$5\cos x \ge 5\sin x - 2$$
$$5(\sin x - \cos x) \le 2$$
$$\sin x - \cos x \le 0.4$$
Draw the horizontal line $y = 0.4$ on the graph of $y = \sin x - \cos x$.
The two graphs intersect at $x = 60°$ and $x = 210°$.
For the ranges $0° \le x \le 60°$ and $210° \le x \le 360°$, the corresponding parts of the graph of $y = \sin x - \cos x$ lie on or below the line $y = 0.4$.
$\therefore$ For $0° \le x \le 360°$, the solutions of $5\cos x \ge 5\sin x - 2$ are $0° \le x \le 60°$ or $210° \le x \le 360°$.

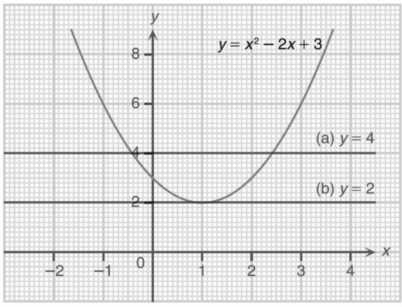
14.



(a) Draw the horizontal line $y = 3$ on the graph of $y = x^3 + 3x^2 - 1$.
The two graphs intersect at $x = -2$ and $x = 1$.
For the range $x > 1$, the corresponding part of the graph of $y = x^3 + 3x^2 - 1$ lies above the line $y = 3$.
$\therefore$ The solutions of $x^3 + 3x^2 - 1 > 3$ are $x > 1$.

(b) Draw the horizontal line $y = -1$ on the graph of $y = x^3 + 3x^2 - 1$.
The two graphs intersect at $x = -3$ and $x = 0$.
When $x \le -3$ and $x = 0$, the corresponding parts of the graph of $y = x^3 + 3x^2 - 1$ lie on or below the line $y = -1$.
$\therefore$ The solutions of $x^3 + 3x^2 - 1 \le -1$ are $x \le -3$ or $x = 0$.

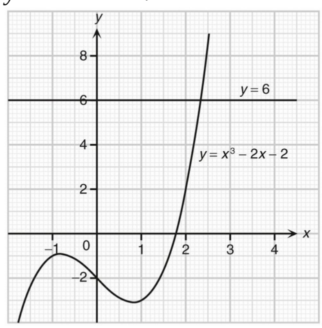
15.



(a)
$$x^2 - 2x - 1 < 0$$
$$x^2 - 2x + 3 < 4$$
Draw the horizontal line $y = 4$ on the graph of $y = x^2 - 2x + 3$.
The two graphs intersect at $x = -0.4$ and $x = 2.4$.
For the range $-0.4 < x < 2.4$, the corresponding part of the graph of $y = x^2 - 2x + 3$ lies below the line $y = 4$.
$\therefore$ The solutions of $x^2 - 2x - 1 < 0$ are $-0.4 < x < 2.4$.

(b)
$$x^2 - 2x + 1 > 0$$
$$x^2 - 2x + 3 > 2$$
Draw the horizontal line $y = 2$ on the graph of $y = x^2 - 2x + 3$.
The two graphs intersect at $x = 1$.
For the ranges $x < 1$ and $x > 1$, the corresponding parts of the graph of $y = x^2 - 2x + 3$ lie above the line $y = 2$.
$\therefore$ The solutions of $x^2 - 2x + 1 > 0$ are all real values of x except $x = 1$.

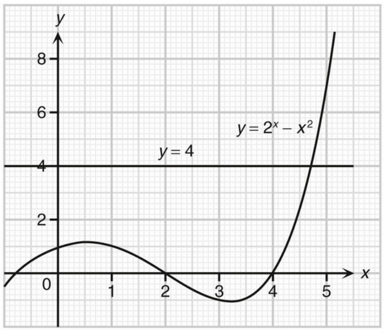
16. (a) Draw the horizontal line $y = 6$ on the graph of $y = x^3 - 2x - 2$.



The two graphs intersect at $x = 2.3$.
For the range $x \ge 2.3$, the corresponding part of the graph of $y = x^3 - 2x - 2$ lies on or above the line $y = 6$.
$\therefore$ The solutions of $x^3 - 2x - 2 \ge 6$ are $x \ge 2.3$.

(b)
$$x^3 \ge 2(x + 4)$$
$$x^3 \ge 2x + 8$$
$$x^3 - 2x - 2 \ge 6$$
From (a), the solutions of $x^3 - 2x - 2 \ge 6$ are $x \ge 2.3$.
$\therefore$ The smallest integer x that satisfies $x^3 \ge 2(x + 4)$ is 3.

17. (a)
$$2^x - x^2 - 4 \le 0$$
$$2^x - x^2 \le 4$$
Draw the horizontal line $y = 4$ on the graph of $y = 2^x - x^2$.

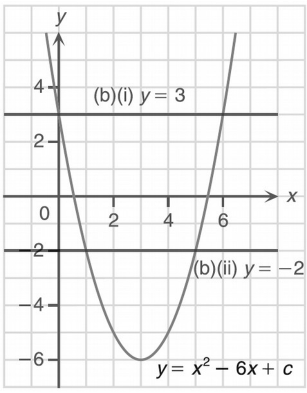


The two graphs intersect at $x = 4.7$.
For the range $x \le 4.7$, the corresponding part of the graph of $y = 2^x - x^2$ lies on or below the line $y = 4$.
∴ The solutions of $2^x - x^2 - 4 \ge 0$ are $x \le 4.7$.

(b)
$$2^x + 4x \le (x+2)^2$$
$$2^x + 4x \le x^2 + 4x + 4$$
$$2^x - x^2 - 4 \le 0$$
From (a), the solutions of $2^x - x^2 - 4 \le 0$ are $x \le 4.7$.
∴ The largest integer x that satisfies $2^x + 4x \le (x + 2)^2$ is 4.

18. (a) ∵ The y-intercept of the graph of $y = x^2 - 6x + c$ is 3.
∴ By substituting (0, 3) into $y = x^2 - 6x + c$, we have
$$3 = 0^2 - 6(0) + c$$
$$c = \underline{\underline{3}}$$

(b)

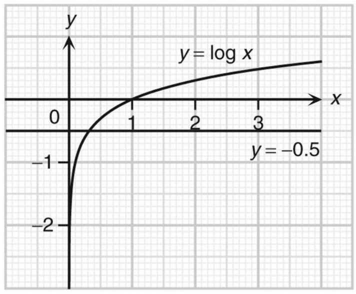


(i)
$$x^2 - 6x \le 0$$
$$x^2 - 6x + 3 \le 3$$
Draw the horizontal line $y = 3$ on the graph of $y = x^2 - 6x + 3$.
The two graphs intersect at $x = 0$ and $x = 6$.
For the range $0 \le x \le 6$, the corresponding part of the graph of $y = x^2 - 6x + 3$ lies on or below the line $y = 3$.
∴ The solutions of $x^2 - 6x \le 0$ are $0 \le x \le 6$.

(ii)
$$-x^2 + 6x \le 5$$
$$x^2 - 6x \ge -5$$
$$x^2 - 6x + 3 \ge -2$$
Draw the horizontal line $y = -2$ on the graph of $y = x^2 - 6x + 3$.
The two graphs intersect at $x = 1$ and $x = 5$.
For the ranges $x \le 1$ and $x \ge 5$, the corresponding parts of the graph of $y = x^2 - 6x + 3$ lie on or above the line $y = -2$.
∴ The solutions of $-x^2 + 6x \le 5$ are $x \le 1$ or $x \ge 5$.

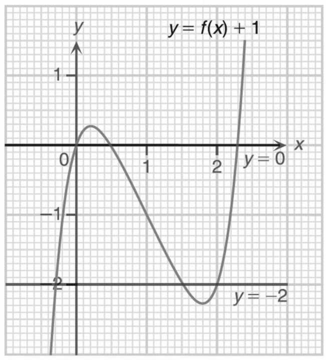
19. (a)
$$\log(1000x^2) = \log x^2 + \log 1000$$
$$= 2\log x + 3$$

(b)
$$\log(1000x^2) > 2$$
$$2\log x + 3 > 2$$
$$2\log x > -1$$
$$\log x > -0.5$$
Draw the horizontal line $y = -0.5$ on the graph of $y = \log x$.



The two graphs intersect at $x = 0.3$.
For the range $x > 0.3$, the corresponding part of the graph of $y = \log x$ lies above the line $y = -0.5$.
∴ The solutions of $\log(1000x^2) > 2$ are $x > 0.3$.

20.
$$-3 < f(x) < -1$$
$$-2 < f(x) + 1 < 0$$



Draw the horizontal line $y = 0$ on the graph of $y = f(x) + 1$.
The two graphs intersect at $x = 0$, $x = 0.5$ and $x = 2.3$.
For the ranges $x < 0$ and $0.5 < x < 2.3$, the corresponding parts of the graph of $y = f(x) + 1$ lie below the line $y = 0$.
∴ The solutions of $f(x) < -1$ are $x < 0$ or $0.5 < x < 2.3$.
Draw the horizontal line $y = -2$ on the graph of $y = f(x) + 1$.
The two graphs intersect at $x = -0.3$, $x = 1.5$ and $x = 2$.
For the ranges $-0.3 < x < 1.5$ or $x > 2$, the corresponding parts of the graph of $y = f(x) + 1$ lie above the line $y = -2$.
∴ The solutions of $f(x) > -3$ are $-0.3 < x < 1.5$ or $x > 2$.
∴ The solutions of $-3 < f(x) < -1$ are $-0.3 < x < 0$, $0.5 < x < 1.5$ or $2 < x < 2.3$.

Exercise 3D (p. 3.63)
Level 1

1. (a) The graph of $y = f(x)$ is translated downwards by 1 unit.
(b) The graph of $y = f(x)$ is translated rightwards by 3 units.
(c) The graph of $y = f(x)$ is reflected about the y-axis.

(d) The graph of $y = f(x)$ is enlarged along the y-axis to 3 times the original.

(e) The graph of $y = f(x)$ is reduced along the y-axis to $\frac{1}{3}$ times the original.

(f) The graph of $y = f(x)$ is reduced along the x-axis to $\frac{1}{3}$ times the original.

2. The graph of $y = f(x)$ is translated rightwards by 5 units.

3. The graph of $y = f(x)$ is reflected about the x-axis.

4. The graph of $y = f(x)$ is reflected about the y-axis.

5. The graph of $y = f(x)$ is translated upwards by 1 unit.

6. ∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = x^2 - 1$ upwards by 4 units.

∴ $g(x) = x^2 - 1 + 4$

$= \underline{\underline{x^2 + 3}}$

7. ∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = x^2 - 1$ leftwards by 4 units.

∴ $g(x) = (x + 4)^2 - 1$

$= x^2 + 8x + 16 - 1$

$= \underline{\underline{x^2 + 8x + 15}}$

8. ∵ The graph of $y = g(x)$ is obtained by reflecting the graph of $y = x^2 - 1$ about the x-axis.

∴ $g(x) = -(x^2 - 1)$

$= \underline{\underline{-x^2 + 1}}$

9. ∵ The graph of $y = g(x)$ is obtained by enlarging the graph of $y = x^2 - 1$ along the y-axis to 4 times the original.

∴ $g(x) = 4(x^2 - 1)$

$= \underline{\underline{4x^2 - 4}}$

10. (a)

x	0	1	2	3	4
$f(x)$	–4	–3	0	2	3
$g(x)$	<u>–1</u>	<u>0</u>	<u>3</u>	<u>5</u>	<u>6</u>

(b)

x	0	1	2	3	4
$f(x)$	–4	–3	0	2	3
$h(x)$	<u>–16</u>	<u>–12</u>	<u>0</u>	<u>8</u>	<u>12</u>

11. (a)

x	–2	–1	0	1	2
$f(x)$	10	6	2	–1	–5
$g(x)$	<u>–10</u>	<u>–6</u>	<u>–2</u>	<u>1</u>	<u>5</u>

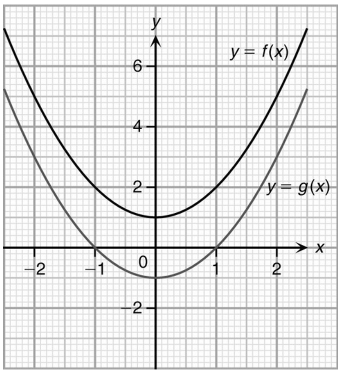
(b)

x	–2	–1	0	1	2
$f(x)$	10	6	2	–1	–5
$h(x)$	<u>–5</u>	<u>–1</u>	<u>2</u>	<u>6</u>	<u>10</u>

12. (a) ∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ downwards by 2 units.

∴ $g(x) = \underline{\underline{f(x) - 2}}$

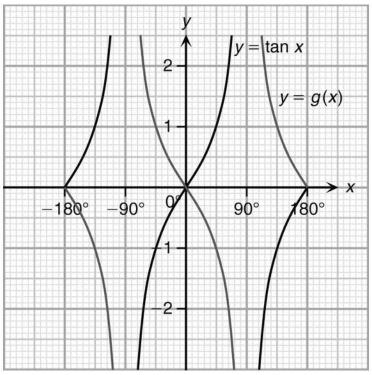
(b)



13. **(a)** ∵ The graph of $y = g(x)$ is obtained by reflecting the graph of $y = \tan x$ about the y-axis.

∴ $g(x) = \underline{\underline{\tan(-x)}}$

(b)



14. **(a)** The graph of $y = 3x^2 - 4x + 2$ is translated downwards by 3 units to give the graph of $y = g(x)$.

(b) $g(x) = (3x^2 - 4x + 2) - 3$
$= \underline{\underline{3x^2 - 4x - 1}}$

15. **(a)** The graph of $y = 2^x$ is enlarged along the y-axis to 5 times the original to give the graph of $y = g(x)$.

(b) $g(x) = \underline{\underline{5(2^x)}}$

16. **(a)** $g(x) = (3^x)^4$
$= 3^{4x}$
$= \underline{\underline{f(4x)}}$

(b) The graph of $y = f(x)$ is reduced along the x-axis to $\frac{1}{4}$ times the original to give the graph of $y = g(x)$.

Level 2

17. $g(x) = x^2 + 4x + 2$
$= (x^2 + 4x - 2) + 4$
$= f(x) + 4$

∴ The graph of $y = f(x)$ is translated upwards by 4 units to give the graph of $y = g(x)$.

18. $g(x) = x^2 + 10x + 25$
$= (x + 5)^2$
$= f(x + 5)$

∴ The graph of $y = f(x)$ is translated leftwards by 5 units to give the graph of $y = g(x)$.

19. $g(x) = 5^{-x}$
$= f(-x)$

∴ The graph of $y = f(x)$ is reflected about the y-axis to give the graph of $y = g(x)$.

20. $g(x) = \log 3x$
$= f(3x)$

∴ The graph of $y = f(x)$ is reduced along the x-axis to $\frac{1}{3}$ times the original to give the graph of $y = g(x)$.

21. $g(x) = \sin x$
$= \sin\left[2\left(\frac{x}{2}\right)\right]$
$= f\left(\frac{x}{2}\right)$

∴ The graph of $y = f(x)$ is enlarged along the x-axis to 2 times the original to give the graph of $y = g(x)$.

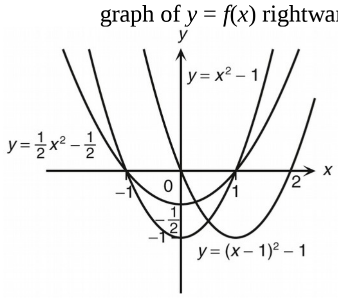
22. Let $f(x) = x^2 - 1$, $g(x) = \frac{1}{2}x^2 - \frac{1}{2}$ and $h(x) = (x - 1)^2 - 1$.

(a) $g(x) = \frac{1}{2}x^2 - \frac{1}{2}$
$= \frac{1}{2}(x^2 - 1)$
$= \frac{1}{2}f(x)$

∴ The graph of $y = g(x)$ is obtained by reducing the graph of $y = f(x)$ along the y-axis to $\frac{1}{2}$ times the original.

(b) $h(x) = (x - 1)^2 - 1$
$= f(x - 1)$

∴ The graph of $y = h(x)$ is obtained by translating the graph of $y = f(x)$ rightwards by 1 unit.



23. **(a)** $f(x) = x^2 - 2x$
$= \underline{\underline{x(x - 2)}}$

$$\begin{aligned} g(x) &= x^2 - 8x + 15 \\ &= \underline{\underline{(x-3)(x-5)}} \end{aligned}$$

(b)
$$\begin{aligned} g(x) &= (x-3)(x-5) \\ &= (x-3)[(x-3)-2] \\ &= f(x-3) \end{aligned}$$

∴ The graph of $y = f(x)$ is translated rightwards by 3 units to give the graph of $y = g(x)$.

24.
$$\begin{aligned} g(x) &= -3^{x+1} + 6 \\ &= 3(-3^x) + 3(2) \\ &= 3(-3^x + 2) \\ &= 3f(x) \end{aligned}$$

∴ The graph of $y = f(x)$ is enlarged along the y-axis to 3 times the original to give the graph of $y = g(x)$.

Alternative Solution

$$\begin{aligned} g(x) &= -3^{x+1} + 6 \\ &= (-3^{x+1} + 2) + 4 \\ &= f(x+1) + 4 \end{aligned}$$

∴ The graph of $y = f(x)$ is translated leftwards by 1 unit and then upwards by 4 units to give the graph of $y = g(x)$.

25. **(a)** ∵ The vertices of the graphs of $y = x^2 - 2x + 5$ and $y = g(x)$ are (1, 4) and (5, 0) respectively.

∴ The graph of $y = x^2 - 2x + 5$ is translated rightwards by 4 units and then downwards by 4 units to give the graph of $y = g(x)$.

(b) Let the graph of $y = h(x)$ be the graph obtained by translating the graph of $y = x^2 - 2x + 5$ rightwards by 4 units.

∴
$$\begin{aligned} h(x) &= (x-4)^2 - 2(x-4) + 5 \\ &= x^2 - 8x + 16 - 2x + 8 + 5 \\ &= x^2 - 10x + 29 \end{aligned}$$

∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = h(x)$ downwards by 4 units.

∴
$$\begin{aligned} g(x) &= h(x) - 4 \\ &= x^2 - 10x + 29 - 4 \\ &= \underline{\underline{x^2 - 10x + 25}} \end{aligned}$$

26. **(a)** The graph of $y = f(x)$ is translated leftwards by 3 units, and then enlarged along the y-axis to 2 times the original to give the graph of $y = g(x)$.

(b) The graph of $y = f(x)$ is reflected about the y-axis, and then reduced along the x-axis to $\frac{1}{3}$ times the original to give the graph of $y = g(x)$.

27.
$$\begin{aligned} h(x) &= 4x^2 + 4x + 2 \\ &= 2(2x^2 + 2x + 1) \\ &= 2[2(-x)^2 - 2(-x) + 1] \\ &= 2f(-x) \end{aligned}$$

∴ The graph of $y = f(x)$ is reflected about the y-axis, and then enlarged along the y-axis to 2 times the original to give the graph of $y = h(x)$.

28. (a) Let C be the image of A when the graph of $y = f(x)$ is reduced along the y-axis to $\frac{1}{2}$ times the original.

$\therefore \quad C = \left(4, \frac{-4}{2}\right)$
$= (4, -2)$

$\because$ B is obtained by translating C leftwards by 1 unit.

$\therefore \quad B = (4-1, -2)$
$= \underline{\underline{(3, -2)}}$

(b) $\because$ $A(4, -4)$ is a point on the graph of $y = f(x)$.

$\therefore$ By substituting $(4, -4)$ into $y = a(x-3)^2 - 5$, we have

$-4 = a(4-3)^2 - 5$
$-4 = a - 5$
$a = \underline{\underline{1}}$

Let the graph of $y = h(x)$ be the graph obtained by reducing the graph of $y = f(x)$ along the y-axis to $\frac{1}{2}$ times the original.

$\therefore \quad h(x) = \frac{1}{2} f(x)$

$\because$ The graph of $y = g(x)$ is obtained by translating the graph of $y = h(x)$ leftwards by 1 unit.

$\therefore$

$$g(x) = h(x+1)$$
$$= \frac{1}{2} f(x+1)$$
$$= \frac{1}{2}\{[(x+1)-3]^2 - 5\}$$
$$= \frac{1}{2}[(x-2)^2 - 5]$$
$$= \underline{\underline{\frac{1}{2}(x-2)^2 - \frac{5}{2}}}$$

29. (a) Let the graph of $y = h(x)$ be the graph obtained by enlarging the graph of $y = (x-3)^2 - 5$ along the y-axis to 2 times the original.

$\therefore \quad h(x) = 2[(x-3)^2 - 5]$
$= 2(x-3)^2 - 10$

$\because$ The graph of $y = g(x)$ is obtained by translating the graph of $y = h(x)$ leftwards by 1 unit.

$\therefore$

$$g(x) = h(x+1)$$
$$= 2[(x+1)-3]^2 - 10$$
$$= \underline{\underline{2(x-2)^2 - 10}}$$

(b) The coordinates of the vertex of the graph of $y = g(x)$ are $(2, -10)$.

30. (a) Let the graph of $y = h(x)$ be the graph obtained by reducing the graph of $y = \cos x$ along the x-axis to $\frac{1}{3}$ times the original.

$\therefore$ $h(x) = \cos 3x$

$\because$ The graph of $y = g(x)$ is obtained by translating the graph of $y = h(x)$ upwards by 3 units.

$\therefore \quad g(x) = h(x) + 3$
$= \underline{\underline{\cos 3x + 3}}$

(b) From (a), $g(x) = \cos 3x + 3$

$\because \quad -1 \le \cos 3x \le 1$

$\therefore$ The maximum value of $g(x)$ $= 1 + 3$
$= \underline{\underline{4}}$

The minimum value of $g(x)$ $= -1 + 3$
$= \underline{\underline{2}}$

$\because$ The period of $\cos x$ is 360°.

$\therefore$ The period of the function $g(x)$

$$= \frac{360°}{3}$$
$$= \underline{\underline{120°}}$$

31. (a) The graph of $y = \cos x + \sin x$ is reflected about the x-axis, and then reduced along the y-axis to $\frac{1}{2}$ times the original to give the graph of $y = g(x)$.

(b) Let the graph of $y = h(x)$ be the graph obtained by reflecting the graph of $y = \cos x + \sin x$ about the x-axis.

$\therefore \quad h(x) = -(\cos x + \sin x)$
$= -\cos x - \sin x$

$\because$ The graph of $y = g(x)$ is obtained by reducing the graph of $y = h(x)$ along the y-axis to $\frac{1}{2}$ times the original.

$\therefore$

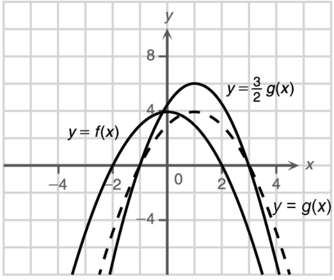
$$g(x) = \frac{1}{2} h(x)$$
$$= \frac{1}{2}(-\cos x - \sin x)$$
$$= \underline{\underline{-0.5\cos x - 0.5\sin x}}$$

32. (a) $g(x) = -(x-1)^2 + 4$
$= f(x-1)$

$\therefore$ The graph of $y = f(x)$ is translated rightwards by 1 unit to give the graph of $y = g(x)$.

(b) From (a), the graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ rightwards by 1 unit.

The graph of $y = \frac{3}{2} g(x)$ is obtained by enlarging the graph of $y = g(x)$ along the y-axis to $\frac{3}{2}$ times the original.



33. (a)

$$g(x) = x^2 + 2x$$
$$= (-x)^2 - 2(-x)$$
$$= f(-x)$$

∴ The graph of $y = f(x)$ is reflected about the y-axis to give the graph of $y = g(x)$.

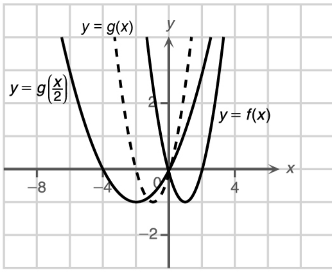
Alternative Solution

$$g(x) = x^2 + 2x$$
$$= x(x+2)$$
$$= (x+2)[(x+2) - 2]$$
$$= (x+2)^2 - 2(x+2)$$
$$= f(x+2)$$

∴ The graph of $y = f(x)$ is translated leftwards by 2 units to give the graph of $y = g(x)$.

(b) From (a), the graph of $y = g(x)$ is obtained by reflecting the graph of $y = f(x)$ about the y-axis.

The graph of $y = g\left(\frac{x}{2}\right)$ is obtained by enlarging the graph of $y = g(x)$ along the x-axis to 2 times the original.



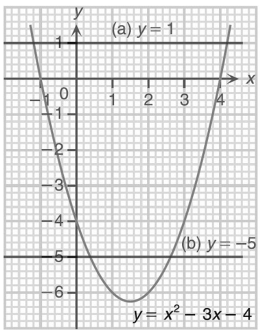
Check Yourself (p. 3.72)

1. **(a)** ✓ **(b)** ✗
(c) ✗ **(d)** ✓

2. The quadratic graph $y = x^2 - 2x - 8$ has reflectional symmetry about the line <u>$x = 1$</u>. It opens <u>upwards</u> and the coordinates of its vertex are <u>(1, −9)</u>.

3. In the figure, graph <u>I</u> may represent the graph of $y = 2^x$ and graph <u>IV</u> may represent the graph of $y = \log_{\frac{1}{2}} x$.

4.



(a)

$$x^2 - 3x - 5 = 0$$
$$x^2 - 3x - 4 = 1$$

Draw the horizontal line $y = 1$ on the graph of $y = x^2 - 3x - 4$.

∵ The two graphs intersect at $x = -1.2$ and $x = 4.2$.

∴ The solutions of $x^2 - 3x - 5 = 0$ are $x = -1.2$ or $x = 4.2$.

(b) $$x^2 - 3x < -1$$
$$x^2 - 3x - 4 < -5$$
Draw the horizontal line $y = -5$ on the graph of $y = x^2 - 3x - 4$.
The two graphs intersect at $x = 0.4$ and $x = 2.6$.
For the range $0.4 < x < 2.6$, the corresponding part of the graph of $y = x^2 - 3x - 4$ lies below the line $y = -5$.
∴ The solutions of $x^2 - 3x < -1$ are $0.4 < x < 2.6$.

5. **(a)** ∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = x^3 + 2x^2 - 2$ downwards by 2 units.
∴ $$\begin{aligned} g(x) &= (x^3 + 2x^2 - 2) - 2 \\ &= \underline{\underline{x^3 + 2x^2 - 4}} \end{aligned}$$

(b) ∵ The graph of $y = g(x)$ is obtained by reflecting the graph of $y = x^3 + 2x^2 - 2$ about the x-axis.
∴ $$\begin{aligned} g(x) &= -(x^3 + 2x^2 - 2) \\ &= \underline{\underline{-x^3 - 2x^2 + 2}} \end{aligned}$$

(c) ∵ The graph of $y = g(x)$ is obtained by reducing the graph of $y = x^3 + 2x^2 - 2$ along the x-axis to half of the original.
∴ $$\begin{aligned} g(x) &= (2x)^3 + 2(2x)^2 - 2 \\ &= \underline{\underline{8x^3 + 8x^2 - 2}} \end{aligned}$$

6. **(a)** $$\begin{aligned} g(x) &= 2(\sin x + 1) \\ &= 2\sin x + 2 \\ &= f(x) + 2 \end{aligned}$$
∴ The graph of $y = f(x)$ is translated upwards by 2 units to give the graph of $y = g(x)$.

(b) $$\begin{aligned} h(x) &= \sin\frac{x}{2} \\ &= \frac{1}{2}\left(2\sin\frac{x}{2}\right) \\ &= \frac{1}{2}f\left(\frac{x}{2}\right) \end{aligned}$$
∴ The graph of $y = f(x)$ is reduced along the y-axis to to half of the original, and then enlarged along the x-axis to 2 times the original to give the graph of $y = h(x)$.

Revision Exercise 3 (p. 3.73)
Level 1

1. **(a)** When $x = 0$,
$$y = -2(0) + 3 = 3$$
∴ The y-intercept is 3.
When $y = 0$,
$$\begin{aligned} 0 &= -2x + 3 \\ -3 &= -2x \\ x &= \frac{3}{2} \end{aligned}$$
∴ The x-intercept is $\frac{3}{2}$.

(b) When $x = 0$,

$$y = (0)^2 - 6(0) + 5 = 5$$

∴ The y-intercept is 5.

When $y = 0$,

$$0 = x^2 - 6x + 5$$
$$0 = (x - 1)(x - 5)$$
$$x = 1 \text{ or } x = 5$$

∴ The x-intercepts are 1 and 5.

(c) When $x = 0$,

$$y = 2^0 = 1$$

∴ The y-intercept is 1.

∵ The graph of $y = 2^x$ lies above the x-axis.

∴ The graph does not have x-intercepts.

(d) ∵ The graph of $y = \log_3 x$ lies on the right-hand side of the y-axis.

∴ The graph does not have y-intercepts.

When $y = 0$,

$$0 = \log_3 x$$
$$x = 3^0$$
$$= 1$$

∴ The x-intercept is 1.

2. For the graph of $y = (x - 3)^2 - 2$,

the axis of symmetry: $x = 3$

the coordinates of the vertex: (3, –2)

When $x = 0$,

$$y = (0 - 3)^2 - 2 = 7$$

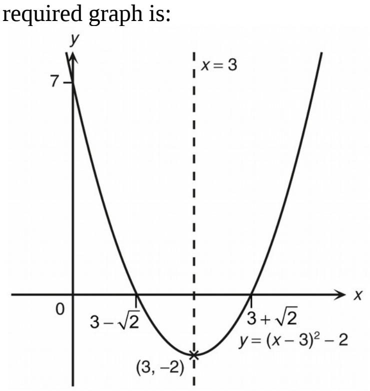
∴ The y-intercept is 7.

When $y = 0$,

$$0 = (x - 3)^2 - 2$$
$$x - 3 = \pm\sqrt{2}$$
$$x = 3 - \sqrt{2} \text{ or } x = 3 + \sqrt{2}$$

∴ The x-intercepts are $3 - \sqrt{2}$ and $3 + \sqrt{2}$.

The required graph is:



3. For the graph of $y = \frac{25}{4} - \left(x - \frac{1}{2}\right)^2$,

the axis of symmetry: $x = \frac{1}{2}$

the coordinates of the vertex: $\left(\frac{1}{2}, \frac{25}{4}\right)$

When $x = 0$,

$$y = \frac{25}{4} - \left(0 - \frac{1}{2}\right)^2 = \frac{25}{4} - \frac{1}{4} = 6$$

∴ The y-intercept is 6.

When $y = 0$,

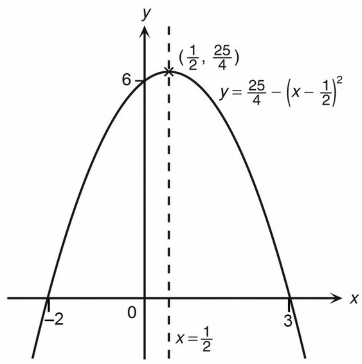
$$0 = \frac{25}{4} - \left(x - \frac{1}{2}\right)^2$$

$$x - \frac{1}{2} = \pm\sqrt{\frac{25}{4}}$$

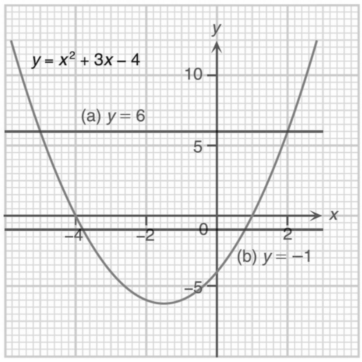
$$x = -2 \quad \text{or} \quad x = 3$$

∴ The x-intercepts are –2 and 3.

The required graph is:



4.



(a) Draw the horizontal line $y = 6$ on the graph of $y = x^2 + 3x - 4$.

∵ The two graphs intersect at $x = -5.0$ and $x = 2.0$.

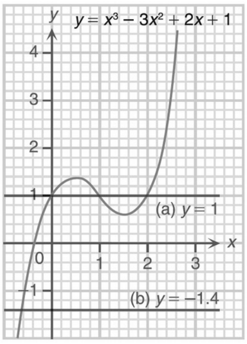
∴ The solutions of $x^2 + 3x - 4 = 6$ are $x = -5.0$ or $x = 2.0$.

(b) Draw the horizontal line $y = -1$ on the graph of $y = x^2 + 3x - 4$.

∵ The two graphs intersect at $x = -3.8$ and $x = 0.8$.

∴ The solutions of $x^2 + 3x - 4 = -1$ are $x = -3.8$ or $x = 0.8$.

5.



(a) Draw the horizontal line $y = 1$ on the graph of $y = x^3 - 3x^2 + 2x + 1$.

∵ The two graphs intersect at $x = 0.0$, $x = 1.0$ and $x = 2.0$.

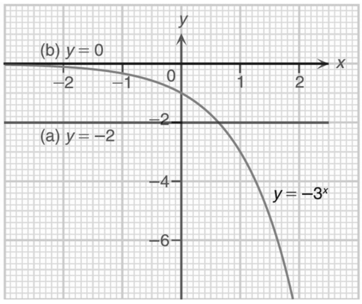
∴ The solutions of $x^3 - 3x^2 + 2x + 1 = 1$ are $x = 0.0$, $x = 1.0$ or $x = 2.0$.

(b) Draw the horizontal line $y = -1.4$ on the graph of $y = x^3 - 3x^2 + 2x + 1$.

∵ The two graphs intersect at $x = -0.6$.

∴ The solution of $x^3 - 3x^2 + 2x + 1 = -1.4$ is $x = -0.6$.

6.



(a) Draw the horizontal line $y = -2$ on the graph of $y = -3^x$.

∵ The two graphs intersect at $x = 0.6$.

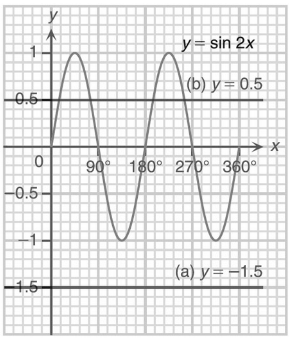
∴ The solution of $-3^x = -2$ is $x = 0.6$.

(b) Draw the horizontal line $y = 0$ on the graph of $y = -3^x$.

∵ The two graphs do not intersect.

∴ $-3^x = 0$ has no real solutions.

7.



(a) Draw the horizontal line $y = -1.5$ on the graph of $y = \sin 2x$.

∵ The two graphs do not intersect.

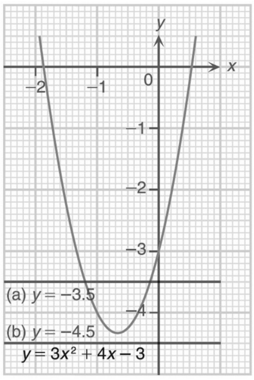
∴ For $0° \le x \le 360°$, $\sin 2x = -1.5$ has no real solutions.

(b) Draw the horizontal line $y = 0.5$ on the graph of $y = \sin 2x$.

∵ The two graphs intersect at $x = 18°$, $x = 72°$, $x = 198°$ and $x = 252°$.

∴ For $0° \le x \le 360°$, the solutions of $\sin 2x = 0.5$ are $x = 18°$, $x = 72°$, $x = 198°$ or $x = 252°$.

8.



(a) Draw the horizontal line $y = -3.5$ on the graph of $y = 3x^2 + 4x - 3$.

The two graphs intersect at $x = -1.2$ and $x = -0.1$.

For the range $-1.2 < x < -0.1$, the corresponding part of the graph of $y = 3x^2 + 4x - 3$ lies below the line $y = -3.5$.

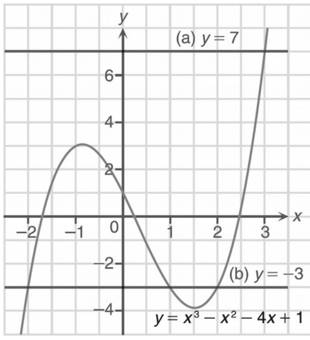
∴ The solutions of $3x^2 + 4x - 3 < -3.5$ are $-1.2 < x < -0.1$.

(b) Draw the horizontal line $y = -4.5$ on the graph of $y = 3x^2 + 4x - 3$.

The two graphs do not intersect, and the whole graph of $y = 3x^2 + 4x - 3$ lies above the line $y = -4.5$.

∴ The solutions of $3x^2 + 4x - 3 > -4.5$ are all real values of x.

9.



(a) Draw the horizontal line $y = 7$ on the graph of $y = x^3 - x^2 - 4x + 1$.

The two graphs intersect at $x = 3$.

For the range $x \ge 3$, the corresponding part of the graph of $y = x^3 - x^2 - 4x + 1$ lies on or above the line $y = 7$.

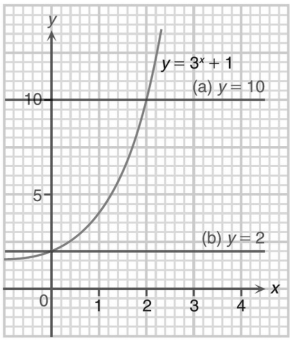
∴ The solutions of $x^3 - x^2 - 4x + 1 \ge 7$ are $x \ge 3$.

(b) Draw the horizontal line $y = -3$ on the graph of $y = x^3 - x^2 - 4x + 1$.

The two graphs intersect at $x = -2$, $x = 1$ and $x = 2$.

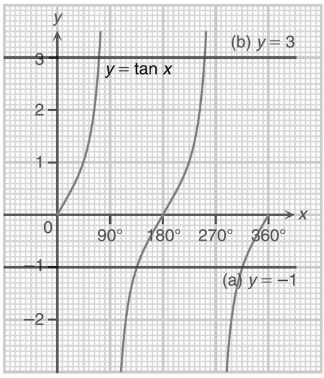
For the ranges $x \le -2$ and $1 \le x \le 2$, the corresponding parts of the graph of $y = x^3 - x^2 - 4x + 1$ lie on or below the line $y = -3$.

∴ The solutions of $x^3 - x^2 - 4x + 1 \le -3$ are $x \le -2$ or $1 \le x \le 2$.

10. 

(a) Draw the horizontal line $y = 10$ on the graph of $y = 3^x + 1$.
The two graphs intersect at $x = 2$.
For the range $x > 2$, the corresponding part of the graph of $y = 3^x + 1$ lies above the line $y = 10$.
∴ The solutions of $3^x + 1 > 10$ are $x > 2$.

(b) Draw the horizontal line $y = 2$ on the graph of $y = 3^x + 1$.
The two graphs intersect at $x = 0$.
For the range $x < 0$, the corresponding part of the graph of $y = 3^x + 1$ lies below the line $y = 2$.
∴ The solutions of $3^x + 1 < 2$ are $x < 0$.

11. 

(a) Draw the horizontal line $y = -1$ on the graph of $y = \tan x$.
The two graphs intersect at $x = 135°$ and $x = 315°$.
For the ranges $90° < x \le 135°$ and $270° < x \le 315°$, the corresponding parts of the graph of $y = \tan x$ lie on or below the line $y = -1$.
∴ For $0° \le x \le 360°$, the solutions of $\tan x \le -1$ are $90° < x \le 135°$ or $270° < x \le 315°$.

(b) Draw the horizontal line $y = 3$ on the graph of $y = \tan x$.
The two graphs intersect at $x = 72°$ and $x = 252°$.
For the ranges $72° \le x < 90°$ and $252° \le x < 270°$, the corresponding parts of the graph of $y = \tan x$ lie on or above the line $y = 3$.
∴ For $0° \le x \le 360°$, the solutions of $\tan x \ge 3$ are $72° \le x < 90°$ or $252° \le x < 270°$.

12. (a) The graph of $y = f(x)$ is translated leftwards by 1 unit to give the graph of $y = g(x)$.

(b) The graph of $y = f(x)$ is translated downwards by 1 unit to give the graph of $y = g(x)$.

13. (a) The graph of $y = f(x)$ is reflected about the y-axis to give the graph of $y = g(x)$.

(b) The graph of $y = f(x)$ is reflected about the x-axis to give the graph of $y = g(x)$.

14. (a) The graph of $y = f(x)$ is enlarged along the x-axis to 2 times the original to give the graph of $y = g(x)$.

(b) The graph of $y = f(x)$ is enlarged along the y-axis to 2 times the original to give the graph of $y = g(x)$.

15. (a) The graph of $y = f(x)$ is reduced along the x-axis to $\frac{1}{3}$ times the original to give the graph of $y = g(x)$.

(b) The graph of $y = f(x)$ is reduced along the y-axis to $\frac{1}{3}$ times the original to give the graph of $y = g(x)$.

16. (a) ∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = x^3$ upwards by 4 units.

∴ $g(x) = \underline{\underline{x^3 + 4}}$

(b) ∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = x^3$ rightwards by 4 units.

∴ $g(x) = \underline{\underline{(x - 4)^3}}$

17. (a) ∵ The graph of $y = g(x)$ is obtained by reflecting the graph of $y = x^3$ about the y-axis.

∴ $g(x) = \underline{\underline{(-x)^3}}$

(b) ∵ The graph of $y = g(x)$ is obtained by enlarging the graph of $y = x^3$ along the y-axis to 2 times the original.

∴ $g(x) = \underline{\underline{2x^3}}$

18. (a) ∵ The graph of $y = g(x)$ is obtained by reducing the graph of $y = x^3$ along the x-axis to $\frac{1}{2}$ times the original.

∴ $g(x) = \underline{\underline{(2x)^3}}$

(b) ∵ The graph of $y = g(x)$ is obtained by enlarging the graph of $y = x^3$ along the x-axis to 2 times the original.

∴ $g(x) = \underline{\underline{\left(\frac{x}{2}\right)^3}}$

19. ∵ From the table, we can observe that $r(k) = h(k) + 9$, where $k = -2, -1, 0, 1$ and 2.

∴ The graph of $y = r(x)$ is obtained by translating the graph of $y = h(x)$ upwards by 9 units.

20. ∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = -x^2 + 2$ leftwards by 2 units.

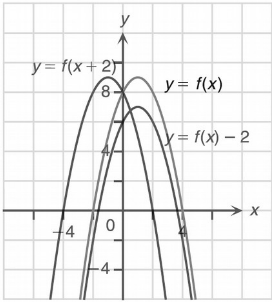
∴ $\underline{\underline{g(x) = -(x+2)^2 + 2}}$ (or $g(x) = -x^2 - 4x - 2$)

21. ∵ The graph of $y = g(x)$ is obtained by reflecting the graph of $y = \log_3 x$ about the x-axis.

∴ $\underline{\underline{g(x) = -\log_3 x}}$

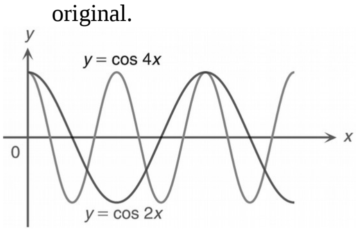
22. **(a)** The graph of $y = f(x) - 2$ is obtained by translating the graph of $y = f(x)$ downwards by 2 units.

(b) The graph of $y = f(x+2)$ is obtained by translating the graph of $y = f(x)$ leftwards by 2 units.



23. ∵ $\cos 2x = \cos\left[\frac{1}{2}(4x)\right]$

∴ The graph of $y = \cos 2x$ is obtained by enlarging the graph of $y = \cos 4x$ along the x-axis to 2 times the original.

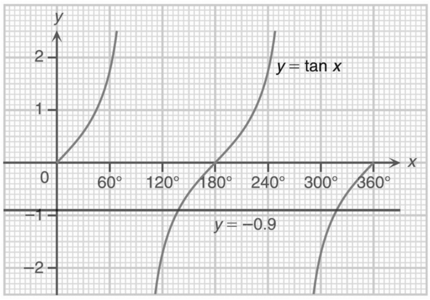


Level 2

24. $\tan x + 0.9 = 0$

$\tan x = -0.9$

Draw the horizontal line $y = -0.9$ on the graph of $y = \tan x$.



∵ The two graphs intersect at $x = 138°$ and $x = 318°$.

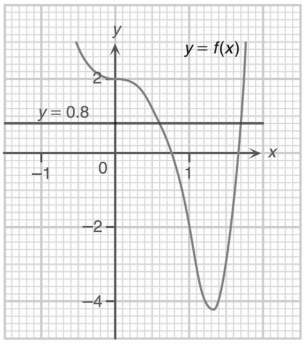
∴ For $0° \le x \le 360°$, the solutions of $\tan x + 0.9 = 0$ are $x = 138°$ or $x = 318°$.

25. $5f(x) - 4 = 0$

$5f(x) = 4$

$f(x) = 0.8$

Draw the horizontal line $y = 0.8$ on the graph of $y = f(x)$.



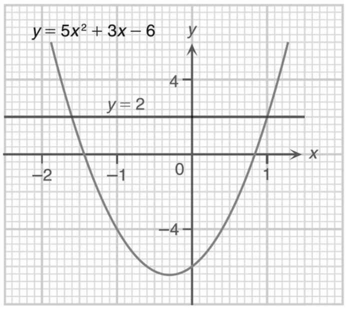
∵ The two graphs intersect at $x = 0.6$ and $x = 1.7$.

∴ The solutions of $5f(x) - 4 = 0$ are $x = 0.6$ or $x = 1.7$.

26. $5x^2 + 3x - 8 = 0$

$5x^2 + 3x - 6 = 2$

Draw the horizontal line $y = 2$ on the graph of $y = 5x^2 + 3x - 6$.



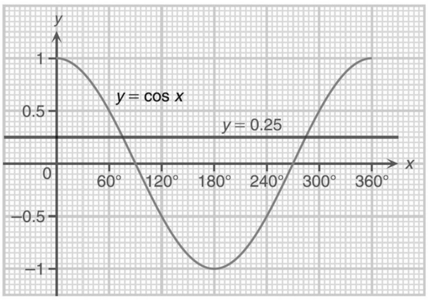
∵ The two graphs intersect at $x = -1.6$ and $x = 1.0$.

∴ The solutions of $5x^2 + 3x - 8 = 0$ are $x = -1.6$ or $x = 1.0$.

27. $4\cos x < 1$

$\cos x < 0.25$

Draw the horizontal line $y = 0.25$ on the graph of $y = \cos x$.



The two graphs intersect at $x = 78°$ and $x = 282°$.

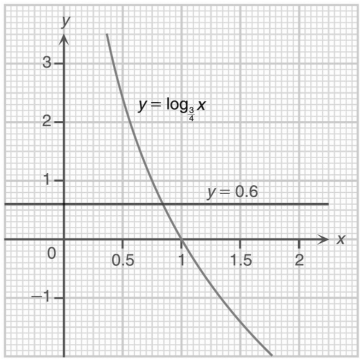
For the range $78° < x < 282°$, the corresponding part of the graph of $y = \cos x$ lies below the line $y = 0.25$.

∴ For $0° \le x \le 360°$, the solutions of $4\cos x < 1$ are $78° < x < 282°$.

28. $5\log_{\frac{3}{4}} x < 3$

$\log_{\frac{3}{4}} x < 0.6$

Draw the horizontal line $y = 0.6$ on the graph of $y = \log_{\frac{3}{4}} x$.



The two graphs intersect at $x = 0.85$.

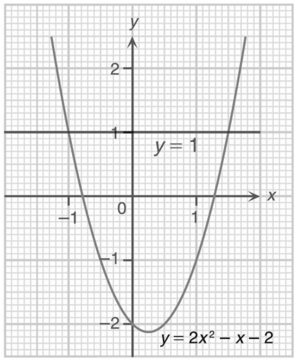
For the range $x > 0.85$, the corresponding part of the graph of $y = \log_{\frac{3}{4}} x$ lies below the line $y = 0.6$.

∴ The solutions of $5\log_{\frac{3}{4}} x < 3$ are $x > 0.85$.

29. $2x^2 - x - 3 > 0$

$2x^2 - x - 2 > 1$

Draw the horizontal line $y = 1$ on the graph of $y = 2x^2 - x - 2$.

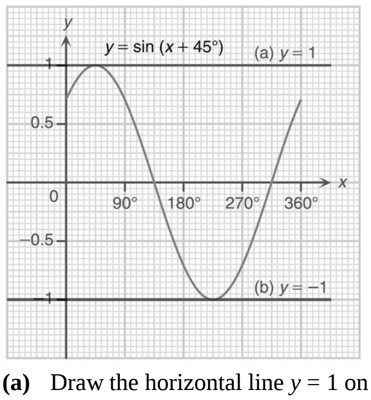


The two graphs intersect at $x = -1$ and $x = 1.5$.

For the ranges $x < -1$ and $x > 1.5$, the corresponding parts of the graph of $y = 2x^2 - x - 2$ lie above the line $y = 1$.

∴ The solutions of $2x^2 - x - 3 > 0$ are $x < -1$ or $x > 1.5$.

30.

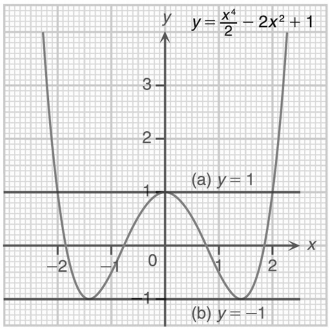


(a) Draw the horizontal line $y = 1$ on the graph of $y = \sin(x + 45°)$.

The two graphs intersect at $x = 45°$.
When $x = 45°$, the corresponding part of the graph of $y = \sin(x + 45°)$ lies on the line $y = 1$.
∴ For $0° \le x \le 360°$, the solution of $\sin(x + 45°) \ge 1$ is $x = 45°$.

(b) Draw the horizontal line $y = -1$ on the graph of $y = \sin(x + 45°)$.
The two graphs intersect at $x = 225°$.
For the ranges $0° \le x < 225°$ and $225° < x \le 360°$, the corresponding parts of the graph of $y = \sin(x + 45°)$ lie above the line $y = -1$.
∴ For $0° \le x \le 360°$, the solutions of $\sin(x + 45°) > -1$ are $0° \le x < 225°$ or $225° < x \le 360°$, i.e. all real values of x except $x = 225°$.

31.



(a) $\frac{x^4}{2} - 2x^2 \ge 0$

$\frac{x^4}{2} - 2x^2 + 1 \ge 1$

Draw the horizontal line $y = 1$ on the graph of $y = \frac{x^4}{2} - 2x^2 + 1$.
The two graphs intersect at $x = -2$, $x = 0$ and $x = 2$.
For the ranges $x \le -2$, $x = 0$ and $x \ge 2$, the corresponding parts of the graph of $y = \frac{x^4}{2} - 2x^2 + 1$ lies on or above the line $y = 1$.
∴ The solutions of $\frac{x^4}{2} - 2x^2 \ge 0$ are $x \le -2$, $x = 0$ or $x \ge 2$.

(b) $\frac{x^4}{2} - 2x^2 + 2 < 0$

$\frac{x^4}{2} - 2x^2 + 1 < -1$

Draw the horizontal line $y = -1$ on the graph of $y = \frac{x^4}{2} - 2x^2 + 1$.
The two graphs intersect at $x = -1.4$ and $x = 1.4$.
The whole graph of $y = \frac{x^4}{2} - 2x^2 + 1$ lies on or above the line $y = -1$.
∴ $\frac{x^4}{2} - 2x^2 + 2 < 0$ has no real solutions.

32.
$$\begin{aligned} g(x) &= (x+3)^2 \\ &= x^2 + 6x + 9 \\ &= (x^2 + 6x + 10) - 1 \\ &= f(x) - 1 \end{aligned}$$

∴ The graph of $y = f(x)$ is translated downwards by 1 unit to give the graph of $y = g(x)$.

33.
$$\begin{aligned} g(x) &= \cos x - \sin^2 x - x^2 \\ &= -(\sin^2 x - \cos x + x^2) \\ &= -f(x) \end{aligned}$$

∴ The graph of $y = f(x)$ is reflected about the x-axis to give the graph of $y = g(x)$.

34.
$$\begin{aligned} g(x) &= \log_2 x^2 \\ &= 2\log_2 x \\ &= 2f(x) \end{aligned}$$

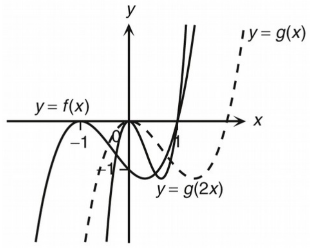
∴ The graph of $y = f(x)$ is enlarged along the y-axis to 2 times the original to give the graph of $y = g(x)$.

35. **(a)** **(i)**
$$\begin{aligned} f(x) &= (x-1)(x^2+2x+1) \\ &= \underline{\underline{(x-1)(x+1)^2}} \end{aligned}$$
$$\begin{aligned} g(x) &= x^3 - 2x^2 \\ &= \underline{\underline{x^2(x-2)}} \end{aligned}$$

(ii)
$$\begin{aligned} g(x) &= x^2(x-2) \\ &= [(x-1)-1][(x-1)+1]^2 \\ &= f(x-1) \end{aligned}$$

∴ The graph of $y = f(x)$ is translated rightwards by 1 unit to give the graph of $y = g(x)$.

(b) From (a)(ii), the graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ rightwards by 1 unit. The graph of $y = g(2x)$ is obtained by reducing the graph of $y = g(x)$ along the x-axis to $\frac{1}{2}$ times the original.



36. **(a)** Let the graph of $y = f(x)$ be the graph obtained by reducing the graph of $y = 2(x+6)^2 + k$ along the y-axis to half of the original.

∴
$$\begin{aligned} f(x) &= \frac{1}{2}[2(x+6)^2 + k] \\ &= (x+6)^2 + \frac{k}{2} \end{aligned}$$

∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = f(x)$ leftwards by 2 units.

∴
$$\begin{aligned} g(x) &= f(x+2) \\ &= [(x+2)^2 + 6]^2 + \frac{k}{2} \\ &= (x+8)^2 + \frac{k}{2} \end{aligned}$$

∴ The vertex of the graph of $y = g(x)$ is $\left(-8, \frac{k}{2}\right)$.

∴ $h = \underline{\underline{-8}}$

and
$$\begin{aligned} -3 &= \frac{k}{2} \\ k &= \underline{\underline{-6}} \end{aligned}$$

(b) From (a), we have
$$\begin{aligned} g(x) &= (x+8)^2 + \frac{-6}{2} \\ &= \underline{\underline{(x+8)^2 - 3}} \end{aligned}$$

37. **(a)** Let the graph of $y = p(x)$ be the graph obtained by reflecting the graph of $y = f(x)$ about the x-axis.

∴ $p(x) = -f(x)$

∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = p(x)$ downwards by 3 units.

∴
$$\begin{aligned} g(x) &= p(x) - 3 \\ &= -f(x) - 3 \\ &= -(x^3 + 2x^2 + 3x + 4) - 3 \\ &= -x^3 - 2x^2 - 3x - 7 \end{aligned}$$

∴ $k = \underline{\underline{7}}$

(b) Let the graph of $y = p(x)$ be the graph obtained by translating the graph of $y = f(x)$ downwards by 3 units.

∴ $p(x) = f(x) - 3$

Let the graph of $y = q(x)$ be the graph obtained by reflecting the graph of $y = p(x)$ about the x-axis.

∴
$$\begin{aligned} q(x) &= -p(x) \\ &= -[f(x) - 3] \\ &= -[(x^3 + 2x^2 + 3x + 4) - 3] \\ &= -x^3 - 2x^2 - 3x - 1 \\ &\neq g(x) \end{aligned}$$

∴ Mary's claim is incorrect.

38. **(a)** Let $f(x) = \log_4 x$ and the graph of $y = h(x)$ be the graph obtained by enlarging the graph of $y = f(x)$ along the x-axis to 4 times the original.

∴ $h(x) = f\left(\frac{x}{4}\right)$

∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = h(x)$ downwards by 2 units.

$$g(x)=h(x)-2$$
$$\therefore \quad =f\left(\frac{x}{4}\right)-2$$
$$=\log_4\frac{x}{4}-2$$

(b)
$$g(x)=\log_4\frac{x}{4}-2$$
$$=\log_4 x-\log_4 4-2$$
$$=\log_4 x-1-2$$
$$=\log_4 x-3$$

∴ The graph of $y = g(x)$ can be obtained by translating the graph of $y=\log_4 x$ downwards by 3 units.

∴ Peter's claim is correct.

39. **(a)** When $y = 0$,

$$x^2-8x+15=0$$
$$(x-3)(x-5)=0$$
$$x=3 \quad \text{or} \quad x=5$$

∴ The x-intercepts of the graph of $y=f(x)$ are 3 and 5.

(b) **(i)** The graph of $y = f(x)$ is translated rightwards by 1 unit to give the graph of $y = g(x)$.

(ii) The graph of $y = g(x)$ is reflected about the y-axis to give the graph of $y = h(x)$.

(c)
$$h(x)=0$$
$$g(-x)=0$$
$$f(-x-1)=0$$

From (a), the x-intercepts of the graph of $y = f(x)$ are 3 and 5.

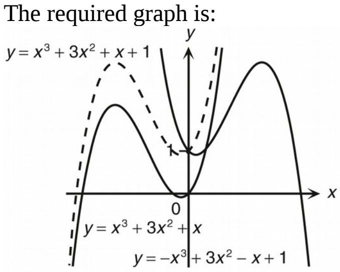
$$\therefore \quad -x-1=3 \quad \text{or} \quad -x-1=5$$
$$x=-4 \quad \text{or} \quad x=-6$$

40. **(a)** Let $f(x)=-x^3+3x^2-x+1$ and $g(x)=x^3+3x^2+x$.

$$g(x)=x^3+3x^2+x$$
$$=(x^3+3x^2+x+1)-1$$
$$=[-(-x)^3+3(-x)^2-(-x)+1]-1$$
$$=f(-x)-1$$

∴ The graph of $y=-x^3+3x^2-x+1$ is reflected about the y-axis, and then translated downwards by 1 unit to give the graph of $y=x^3+3x^2+x$.

(b) The required graph is:



Multiple Choice Questions (p. 3.80)

1. **Answer: B**

Both the functions $y = \sin x$ and $y = \cos x$ have the maximum value 1 and the minimum value −1.

∴ I is true.

The graph of $y = \cos x$ is symmetrical about the y-axis but

the graph of $y = \sin x$ is not symmetrical about the y-axis.
∴ II is not true.

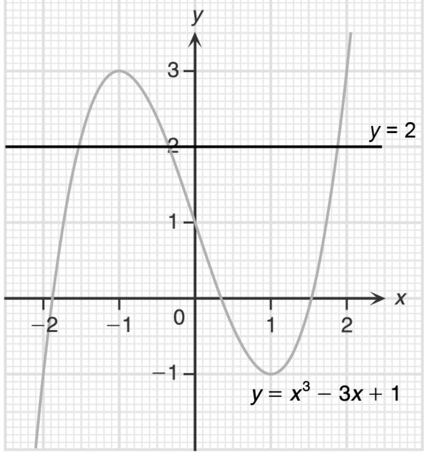
Both the functions $y = \sin x$ and $y = \cos x$ are periodic, and their periods are 360°.
∴ III is true.
∴ The answer is B.

2. Answer: C
For option C, the function $y = 2^x$ does not have minimum value.

3. Answer: B

$$x^3 - 3x - 1 = 0$$
$$x^3 - 3x + 1 = 2$$

Draw the horizontal line $y = 2$ on the graph of $y = x^3 - 3x + 1$.

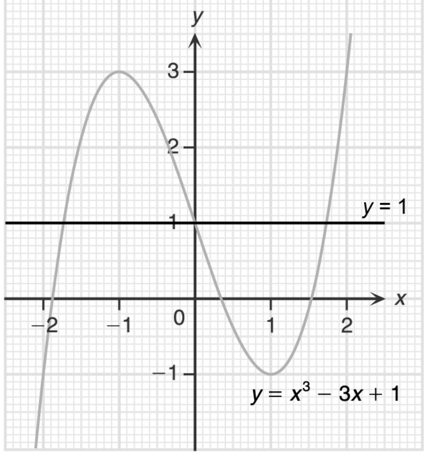


∵ The two graphs intersect at $x = -1.5$, $x = -0.3$ and $x = 1.9$.
∴ The solutions of $x^3 - 3x - 1 = 0$ are $x = -1.5$, $x = -0.3$ or $x = 1.9$.

4. Answer: C

$$x^3 - 3x < 0$$
$$x^3 - 3x + 1 < 1$$

Draw the horizontal line $y = 1$ on the graph of $y = x^3 - 3x + 1$.

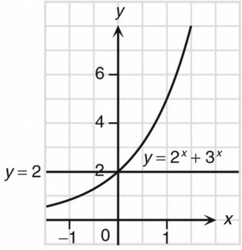


The two graphs intersect at $x = -1.7$, $x = 0$ and $x = 1.7$.
For the ranges $x < -1.7$ and $0 < x < 1.7$, the corresponding parts of the graph of $y = x^3 - 3x + 1$ lie below the line $y = 1$.
∴ The solutions of $x^3 - 3x < 0$ are $x < -1.7$ or $0 < x < 1.7$.

5. **Answer: A**

$$2^x + 3^x - 2 \geq 0$$
$$2^x + 3^x \geq 2$$

Draw the horizontal line $y = 2$ on the graph of $y = 2^x + 3^x$.



The two graphs intersect at $x = 0$.

For the range $x \geq 0$, the corresponding part of the graph of $y = 2^x + 3^x$ lies on or above the line $y = 2$.

∴ The solutions of $2^x + 3^x - 2 \geq 0$ are $x \geq 0$.

6. **Answer: D**

$$-x^2 + 4x + 5 = k$$
$$-x^2 + 4x + 5 - k = 0 \quad \text{......(*)}$$

If (*) has real solution(s), then

$$\Delta \text{ of } (*) \geq 0$$
$$4^2 - 4(-1)(5-k) \geq 0$$
$$4(5-k) \geq -16$$
$$5-k \geq -4$$
$$k \leq 9$$

7. **Answer: C**

For I,

$$-f(x) = -(x^3 - 3x) = 3x - x^3 = g(x)$$

∴ Transformation I on the graph of $y = f(x)$ gives the graph of $y = g(x)$.

For II,

$$f(-x) = (-x)^3 - 3(-x) = -x^3 + 3x = g(x)$$

∴ Transformation II on the graph of $y = f(x)$ gives the graph of $y = g(x)$.

For III,

$$3f(x) = 3(x^3 - 3x) = 3x^3 - 9x \neq g(x)$$

∴ Transformation III on the graph of $y = f(x)$ does not give the graph of $y = g(x)$.

∴ The answer is C.

8. **Answer: D**

For I,

∵ The graph of $y = g(x)$ passes through (–2, 0).

∴ $g(-2) = 0$

∵ $f(-2-4) = f(-6) \neq 0$

∴ $g(x) \neq f(x-4)$

∴ I is not true.

For II,

∵ The graph of $y = g(x)$ can be obtained by translating the graph of $y = f(x)$ leftwards by 4 units.

∴ $g(x) = f(x + 4)$

∴ II is true.

For III,
∵ The graph of $y = g(x)$ can be obtained by reflecting the graph of $y = f(x)$ about the y-axis.
∴ $g(x) = f(-x)$
∴ III is true.
∴ The answer is D.

9. Answer: C
Let the graph of $y = h(x)$ be the graph obtained by enlarging the graph of $y = \sin 2x$ along the x-axis to 2 times the original.

$$\therefore \quad h(x) = \sin 2\left(\frac{x}{2}\right) = \sin x$$

∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = h(x)$ upwards by 1 unit.

$$\therefore \quad g(x) = h(x) + 1 = \underline{\underline{\sin x + 1}}$$

10. Answer: A
Let the graph of $y = h(x)$ be the graph obtained by translating the graph of $y = f(x)$ downwards by 2 units.

$$\therefore \quad h(x) = f(x) - 2$$

∵ The graph of $y = g(x)$ is obtained by reflecting the graph of $y = h(x)$ about the x-axis.

$$\therefore \quad g(x) = -h(x) = -[f(x) - 2] = -(x^3 + 2x - 2) + 2 = -x^3 - 2x + 4$$

$$\therefore \quad k = \underline{\underline{4}}$$

Exam Focus

Exam-type Questions (p. 3.83)

1. Answer: D
The graph of $y = -f(x - 2)$ is obtained by translating the graph of $y = f(x)$ rightwards by 2 units and then reflecting about the x-axis.
∴ The answer is D.

2. Answer: A

$$2g(x) = f(x)$$

$$g(x) = \frac{1}{2} f(x)$$

∴ The graph of $y = g(x)$ is obtained by reducing the graph of $y = f(x)$ along the y-axis to $\frac{1}{2}$ times the original.
From the graph, y-intercept of the graph of $y = f(x)$ is -8.
∴ The y-intercept of the graph of $y = g(x)$ is -4.
∴ A is correct.

3. Answer: C
∵ The vertex of C_2 is below the vertex of C_1.
∴ C_1 must be reduced along the y-axis or translated downwards to give C_2.
∵ The y-coordinate of the vertex of C_1 is positive and the y-coordinate of the vertex of C_2 is 0.
∴ C_1 must be translated downwards to give C_2.
∵ The y-intercepts of C_1 and C_2 are negative, and the y-intercept of C_2 is greater than that of C_1.
∴ C_1 must be reduced along the y-axis to give C_2.
∴ The answer is C.

4. Answer: D
The graph can be obtained by reflecting the graph of $y = \cos x$ about the x-axis, and then reducing along the y-axis to half of the original, and finally enlarging along the x-axis to 3 times the original.
∴ $y = -3\cos 2x$

5. Answer: A
For I:
The graph of $y = f(x)$ has three x-intercepts.
∴ $f(x) = 0$ does not have a unique solution.
∴ I is not true.
For II:
The graph of $y = f(x)$ intersects with the graph of $y = k$ at $x = -3$, $x = -1$ and $x = 3$.
∴ The solutions of $f(x) = k$ are $x = -3$, $x = -1$ or $x = 3$.
∴ II is true.
For III:
For the ranges $x < -3$ and $-1 < x < 3$, the corresponding parts of the graph of $y = f(x)$ lie above the line $y = k$.
∴ The solutions of $f(x) > k$ are $x < -3$ or $-1 < x < 3$.
∴ II is not true.
∴ The answer is A.

6. (a) Let the graph of $y = p(x)$ be the graph obtained by translating the graph of $y = f(x)$ upwards by 2 units.

$$\therefore \quad p(x) = f(x) + 2$$

∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = p(x)$ rightwards by 5 units.

$$\therefore \quad g(x) = p(x - 5) = f(x - 5) + 2 = \underline{\underline{(x - 5)^2 + 2}}$$

(b) Let the graph of $y = q(x)$ be the graph obtained by translating the graph of $y = 3^{f(x)}$ upwards by 2 units.

$$\therefore \quad q(x) = 3^{f(x)} + 2$$

∵ The graph of $y = h(x)$ is obtained by translating the graph of $y = q(x)$ rightwards by 5 units.

$$\therefore \quad h(x) = q(x - 5) = 3^{f(x-5)} + 2 = \underline{\underline{3^{(x-5)^2+1} + 2}}$$

7. (a) Let the graph of $y = h(x)$ be the graph obtained by reducing the graph of $y = f(x)$ along the y-axis to m times the original.

$$\therefore \quad h(x) = mf(x)$$

∵ The graph of $y = g(x)$ is obtained by translating the graph of $y = h(x)$ leftwards by 2 units.

$$\therefore \quad g(x) = h(x + 2) = mf(x + 2) = m\{2[(x + 2) + 6]^2 - 6\} = m[2(x + 8)^2 - 6] = 2m(x^2 + 16x + 64) - 6m = 2mx^2 + 32mx + 122m$$

Also, $g(x) = (x + h)^2 - 3 = x^2 + 2hx + h^2 - 3$

$1 = 2m$

$\therefore \quad m = \dfrac{1}{2}$

$2h = 32m$

and $2h = 32\left(\dfrac{1}{2}\right)$

$h = 8$

(b) From (a), $g(x) = (x+8)^2 - 3$

$\therefore$ The vertex of the graph of $y = g(x)$ is (–8, –3).

$\because$ The new graph is opening upwards with the vertex (–4, –6).

$\therefore$ The new graph is obtained by translating the graph of $y = g(x)$ downwards by 3 units and then rightwards by 4 units.

Investigation Corner (p. 3.88)

(a) Transformation I: The graph of $y = x^2$ is enlarged along the y-axis to 2 times the original to give the graph of $y = 2x^2$.

Transformation II: The graph of $y = 2x^2$ is translated rightwards by 3 units to give the graph of $y = 2(x-3)^2$.

Transformation III: The graph of $y = 2(x-3)^2$ is translated upwards by 4 units to give the graph of $y = 2(x-3)^2 + 4$.

(b) **(i)** **(1)** Reduce the graph of $y = x^2$ along the y-axis to a times the original, and then translate leftwards by $(-h)$ units and upwards by k units.

(2) Reflect the graph of $y = x^2$ about the x-axis, and translate leftwards by $(-h)$ units and upwards by k units.

(3) Enlarge the graph of $y = x^2$ along the y-axis to $(-a)$ times the original, reflect about the x-axis, and translate leftwards by $(-h)$ units and upwards by k units.

(ii) **(1)** Yes, three transformations are enough.

(2) Yes, three transformations are enough.

(3) No, four transformations are needed.